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Electromagnetism and fundamental constants in the forthcoming International System of Units

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Quantities and units

Quantity (VIM 1.1 ↗)

Property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference.

$$X = \{X\} \; [X]$$

- X Quantity;
- $\{X\}$ value;
- $[X]$ unit

Examples

$$L = 3\,\mathrm{m} \Rightarrow \begin{array}{l} \{L\} = 3 \\ [L] = \mathrm{m} \end{array}$$

$$E = 10\,\mathrm{V/m} \Rightarrow \begin{array}{l} \{E\} = 10 \\ [E] = \mathrm{V/m} \end{array}$$

Quantities and units

System of quantities (VIM 1.3 ↗)

Set of quantities together with a set of noncontradictory equations relating those quantities.

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Remarks

- The number of base units is arbitrary: too few base units make dimensional analysis useless; too many, overwhelming.
- A system of units in which each derived unit is a product of powers of base units with no other proportionality factor than one is called *coherent*.

Quantities and units

International System of Quantities, *ISQ* (VIM 1.6 ↗)

System of quantities based on the seven base quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity.

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System of units, based on the International System of Quantities, their names and symbols, including a series of prefixes and their names and symbols, together with rules for their use, adopted by the General Conference on Weights and Measures (CGPM).

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Remarks

- The SI is a coherent system of units.
- The SI is also *rationalized*: Maxwell equations do not contain any 4π factor.

The International System of units (SI

The seven base units

- m The metre is the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second.
- kg The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
- s The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.
- A The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
- K The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
- mol The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12.
- cd The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

The International System of units (SI

many derived units

Examples

$\mathrm{m/s}$ unit of velocity;

$\mathrm{W = kg\,m/s^2}$ unit of power;

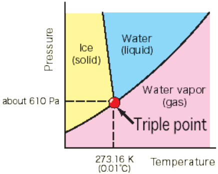
$\mathrm{V = kg m^2 s^{-3} A^{-1}}$ unit of electrical potential difference (voltage)

Definition of units

in the present SI

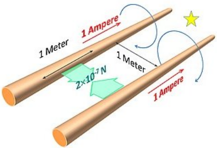
an artefact:

> *The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.*



a natural property

> *The kelvin is the fraction* 1/273.16 *of the thermodynamic temperature of the triple point of water.*



an idealized experiment

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length [. . .] would produce a force equal to* 2×10^{-7} *newton per metre of length*

The realization of the units

Realization (VIM 5.1 ↗)

The realization of the definition of a unit can be provided by a measuring system, a material measure, or a reference material.

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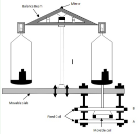
Examples

an artefact:

The international prototype of the kilogram is the realization of the kilogram.

a device

A triple point of water cell is a realization of the kelvin.



an experiment

The current balance is a realization of the ampere.

Defining and realizing units

Reproduction (VIM 5.1 ↗)

The *reproduction* of a unit consists in realizing the unit not from its definition but in setting up a highly reproducible measurement standard based on a physical phenomenon, and, usually, by assigning to it a conventional value.

Defining and realizing units

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Examples

In the present SI:

- The volt is reproduced by means of the Josephson effect.
- The ohm is reproduced by means of the quantum Hall effect.
- The thermodynamic temperature scale is reproduced through two conventional temperature scales, the *International Temperature Scale of 1990* (ITS-90) and the *Provisional Low Temperature Scale of 2000* (PLTS-2000).

SI units for electromagnetic quantities

Base units

Symbol	Unit name
m	metre
kg	kilogram
s	second
A	ampere

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Derived units

$\mathrm{m}^{\alpha}\,\mathrm{kg}^{\beta}\,\mathrm{s}^{\gamma}\,\mathrm{A}^{\delta}$, where α, β, γ, and δ are usually integers.

SI units for electromagnetic quantities

Derived units with special names

Derived quantity	name	symbol	expression in terms of base units
frequency	hertz	Hz	s^{-1}
energy	joule	J	$m^2\,kg\,s^{-2}$
power	watt	W	$m^2\,kg\,s^{-3}$
electric charge	coulomb	C	$s\,A$
electric potential difference	volt	V	$m^2\,kg\,s^{-3}\,A^{-1}$
electric capacitance	farad	F	$m^{-2}\,kg^{-1}\,s^{-4}\,A^2$
electric resistance	ohm	Ω	$m^2\,kg\,s^{-3}\,A^{-2}$
electric conductance	siemens	S	$m^{-2}\,kg^{-1}\,s^3\,A^2$
magnetic flux	weber	Wb	$m^2\,kg\,s^{-2}\,A^{-1}$
magnetic flux density	tesla	T	$kg\,s^{-2}\,A^{-1}$
inductance	henry	H	$m^2\,kg\,s^{-2}\,A^{-2}$

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Remark

Can form further derived units. For example, permittivity can be expressed either in F/m or in $m^{-3}\,kg^{-1}\,s^4\,A^{-2}$.

SI prefixes and suffixes

The SI adopts a series of prefix names and prefix symbols to form the names and symbols of the decimal multiples and submultiples of units, ranging from 10^{24} to 10^{-24}.

name	symbol	factor	name	symbol	factor
yocto	y	10^{-24}	deca	da	10^{1}
zepto	z	10^{-21}	hecto	h	10^{2}
atto	a	10^{-18}	kilo	k	10^{3}
femto	f	10^{-15}	mega	M	10^{6}
pico	p	10^{-12}	giga	G	10^{9}
nano	n	10^{-9}	tera	T	10^{12}
micro	μ	10^{-6}	peta	P	10^{15}
milli	m	10^{-3}	exa	E	10^{18}
centi	c	10^{-2}	zetta	Z	10^{21}
deci	d	10^{-1}	yotta	Y	10^{24}

The expression of the value of electromagnetic quantities benefits of large or small prefixes, more often than in other scientific fields. For example, it is common to speak of fA current, PΩ resistance, or aF capacitance values.

The ampere

In the present SI, the definition of the base unit ampere is mechanical:

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.*

All electromagnetic derived units have an ultimately mechanical definition also.

These quantities are **exact**:

$$\mu_0 = 4 \times 10^{-7}\ \mathrm{H/m} \text{ the } \textit{magnetic constant};$$

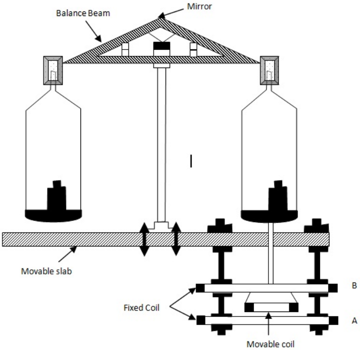
$$\epsilon_0 = \left(\mu_0 c^2\right)^{-1} = 8.854\,187\,817\ldots\ \mathrm{pF/m}, \text{ the } \textit{electric constant}$$

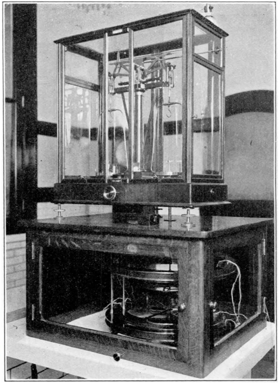
$$Z_0 = \mu_0\, c = \sqrt{\mu_0\, \epsilon_0^{-1}} = 376.730\,313\,4\ldots\ \Omega, \text{ the } \textit{impedance of free space}$$

μ_0, ϵ_0 constant ⇒ realization of SI units of impedance.

Realization of the ampere

The (electrodynamic) ampere balance (Vigoreux, 1965)





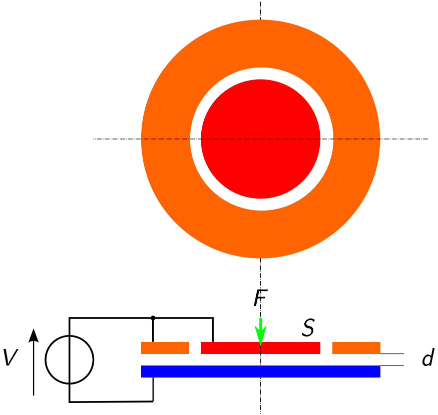
Ampère force law:

$$F = \frac{\mu_0}{4\pi} \int_{\Gamma_1} \int_{\Gamma_2} \frac{I_1 \, \mathrm{d}\boldsymbol{\ell}_1 \times I_2 \, \mathrm{d}\boldsymbol{\ell}_2 \times \boldsymbol{r}_{21}}{|\boldsymbol{r}_{21}|^2}$$

If $I_1 = I_2$, $F = \mu_0 k I^2$ where k is computed from geometrical measurements

Realization of the volt

The (electrostatic) voltage balance



Force between plates: $F = \epsilon_0 \dfrac{S}{2d^2} V^2 = \epsilon_0\, k\, V^2$

where k is computed from geometrical measurements

Realization of the volt

Cylindrical-electrode voltage balance, PTB (Siencknecht and Funck, 1986)

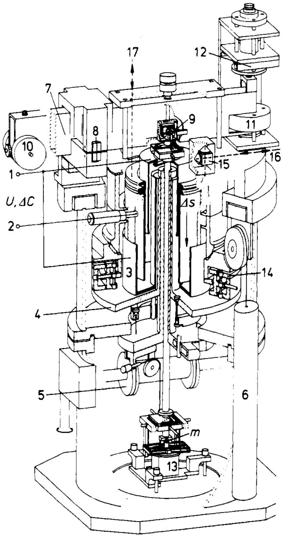
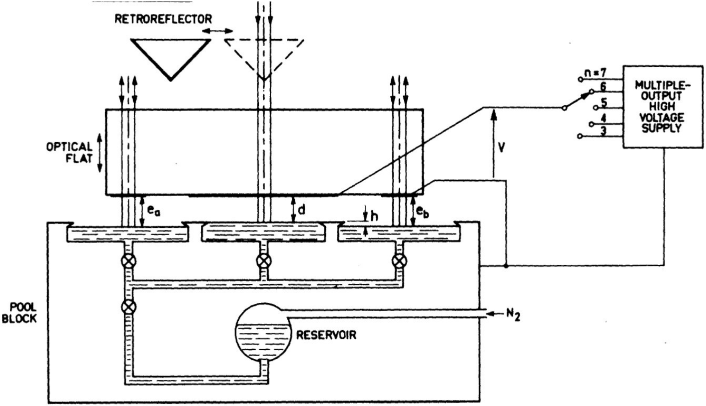


Fig. 1. Perspective view of the PTB voltage balance. 1 Inner electrode, 2 high-voltage electrode, 3 guard electrode, 4 carriage of displace unit, 5 driving device for displace unit, 6 counterweight of displace unit, 7 balance beam, 8 central joint of balance beam, 9 load joint of balance beam, 10 counterbalance weight, 11 position sensor, 12 retainer for balance beam, 13 load-changing device, 14 device for centering and vertical electrode adjustment, 15 interferometer for Δs-measurement, 16 light beam of interferometers for Δs-measurement, 17 light beam of autocollimator for vertical electrode adjustment

$$V = 10\,186\,\mathrm{V} = 1000 \times E_{\mathrm{Weston}};\ m = 2\,\mathrm{g}\ !$$

Realization of the volt

Mercury-electrode elevation, CSIRO Australia (Sloggett et al., 1985)

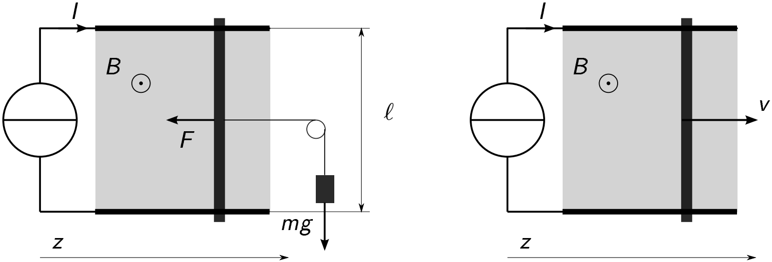


$$V = \sqrt{\frac{2\rho g}{\epsilon_0}} d\sqrt{h}.\ V = \mathrm{kV},\ d = 600\,\mu\mathrm{m},\ u_V = 0.33 \times 10^{-6}$$

Realization of the electrical watt

The watt balance, or Kibble balance

Solves the problem of geometrical measurements!



- Weighing mode: $F = B\ell I = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z} I$
- Moving mode: $E = \dfrac{\mathrm{d}\Phi}{\mathrm{d}t} = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z}\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z} v$
- $Fv = EI$; $P_\mathrm{m} = P_\mathrm{e}$

The Kibble balance

(Robinson and Schlamminger, 2016)

Solves the problem of geometrical measurements!

weighing mode

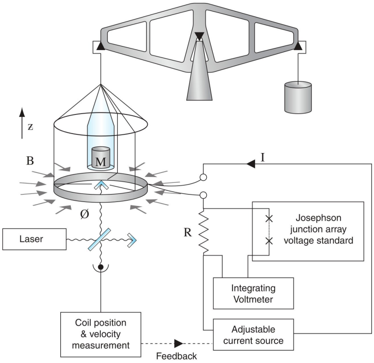


Figure 1. The Kibble balance in weighing mode.

moving mode

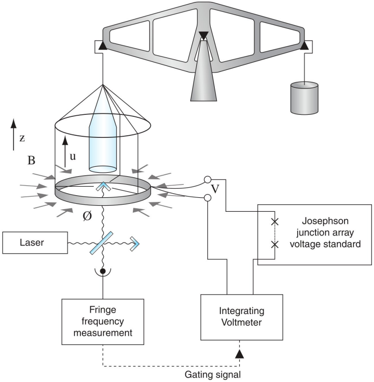
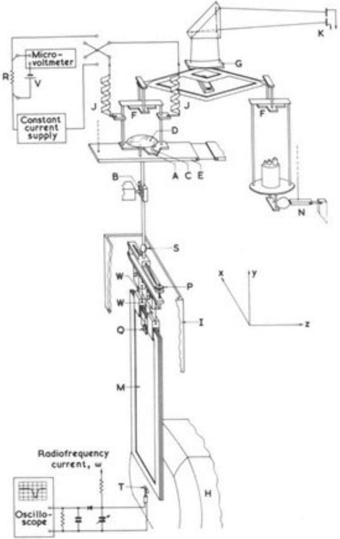


Figure 2. The Kibble balance in moving mode.

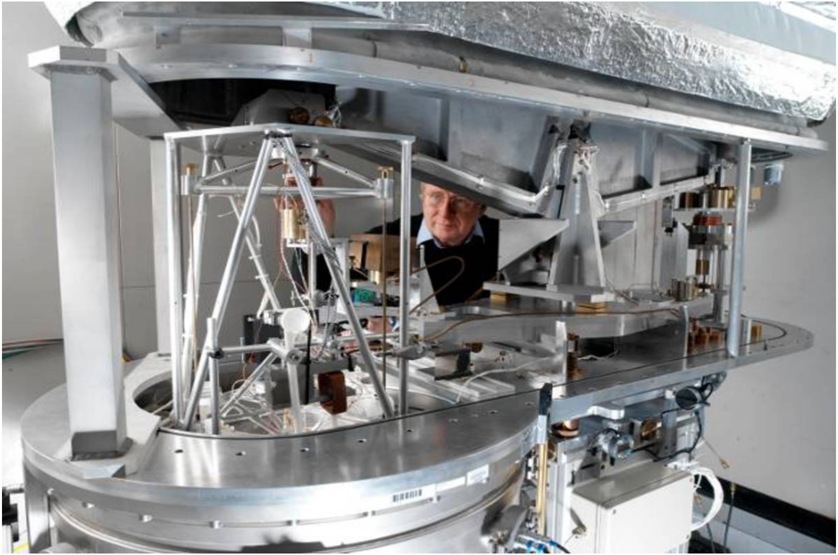
The Kibble balance evolution

NPL, Kibble (1976) for the gyromagnetic ratio of the proton



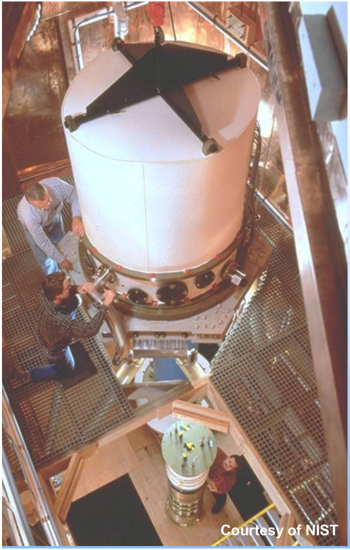
The Kibble balance: evolution

NRC, Bryan P. Kibble and I. Robinson, 2011

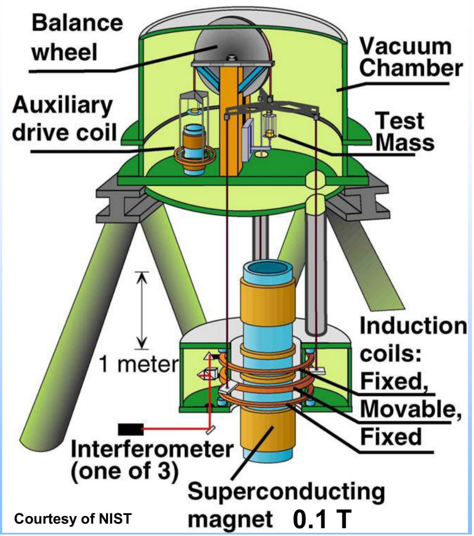


The Kibble balance: evolution

NIST-3



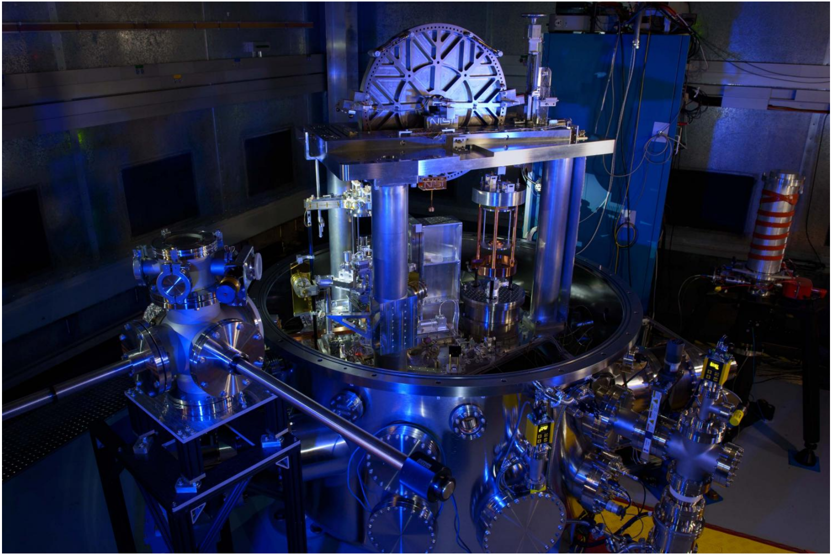
Courtesy of NIST



Courtesy of NIST

The Kibble balance: evolution

The next generation: NIST-4, 2016



The Kibble balance: evolution

The next generation: NPL, 2017

The Kibble balance

Determination of the Planck constant

- $mgv = EI$
- $E = n\frac{f_{\mathrm{E}}}{K_{\mathrm{J}}}$
- $I = \frac{V_{\mathrm{I}}}{R} = \frac{f_{\mathrm{I}}}{K_{\mathrm{J}}}\frac{1}{rR_{\mathrm{K}}}$
- $K_{\mathrm{J}} = \frac{2e}{h}$
- $R_{\mathrm{K}} = \frac{h}{e^2}$

$$\Rightarrow mgv = hf_{\mathrm{E}}f_{\mathrm{I}}\frac{n}{r}$$

h can be measured mechanically

Realization of impedance units

Calculable geometries

if the geometry of the system of conductors is sufficiently simple, explicit mathematical expressions for their inductance or capacitance value may exist. For example:

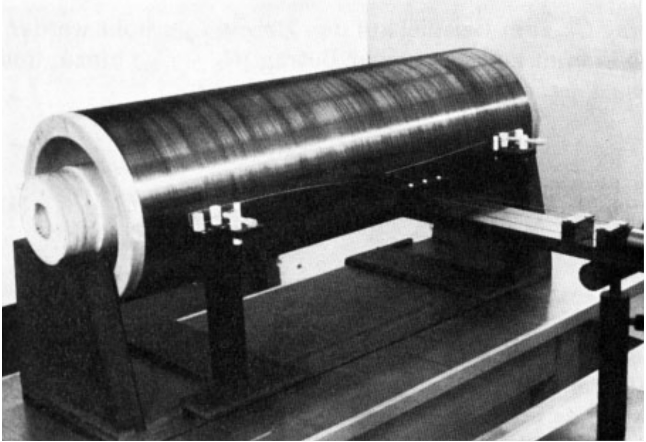
- the low-frequency inductance L of a circular conductive loop (of radius r), made of a circular perfect conductor (of radius a), in vacuum, is $L = \mu_0 \, r \, [\log(8r/a) - 7/4]$;
- the capacitance C of a conducting sphere of radius R in vacuum is $C = 4\pi\epsilon_0 R$.

The previous examples are not adequate for a practical impedance realization, which require a careful choice of the calculable geometrical shape of conductors in order to minimize:

- the dependence of L or C on inevitable deviations of the mechanical realization of conductors' shapes from the ideal geometry employed in the mathematical modelling;
- the number, and practical difficulty, of the accurate length measurements which are needed in the calculation.

Realization of impedance units: the henry

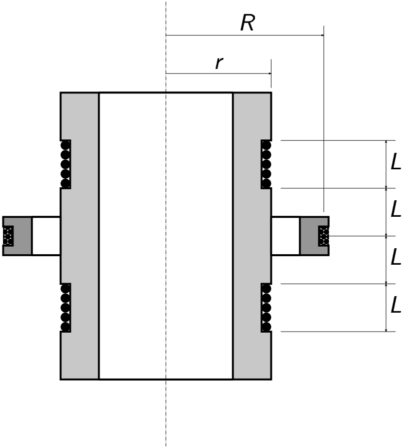
The PTB self-inductor (Linckh and Brasack, 1968)

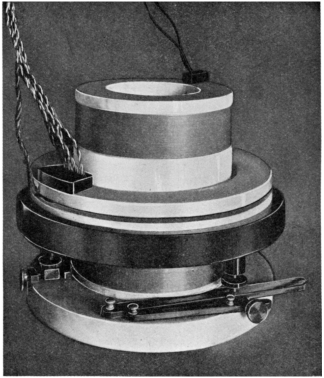


$L = \mu_0 k N^2$ where k is determined by geometrical measurements

Realization of the inductance unit, the henry

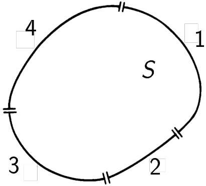
The NPL Mutual inductor (Campbell, 1907)





Realization of capacitance unit, the farad

the calculable capacitor



The general geometry of four conductors 1, 2, 3, 4 having cylindrical symmetry, and arranged in a closed shell with infinitesimal gaps, analyzed by the Thompson-Lampard theorem.

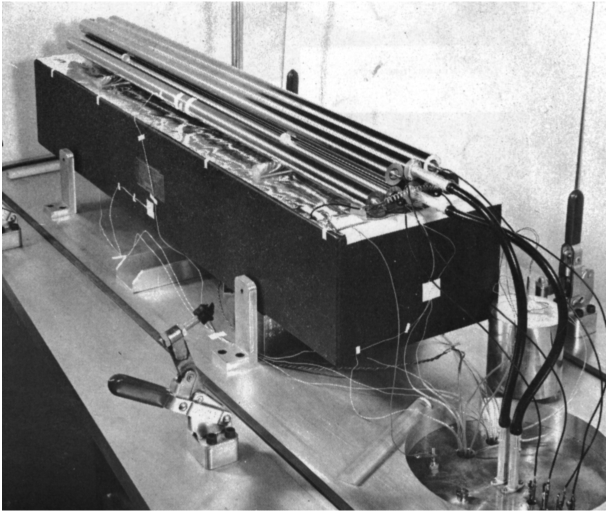
Thompson-Lampard theorem (Lampard, 1957)

$$\exp\left(-\pi\epsilon_0 C_{13}\right) + \exp\left(-\pi\epsilon_0 C_{24}\right) = 1.$$

If there is sufficient symmetry such that $C_{13} = C_{24} = C$,

$$C = \epsilon_0 \frac{\log 2}{\pi} = 1.953549043\ldots \times 10^{-12}\ \mathrm{F/m} \qquad \text{[exact]}.$$

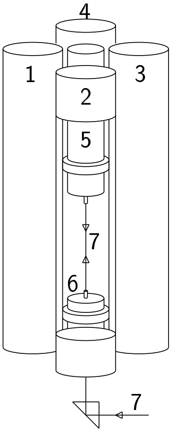
The calculable capacitor



1964: Fixed calculable capacitor, realized with stacked gauge bars, NRC (Dunn, 1964).

Realization of capacitance unit, the farad

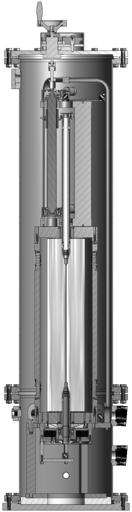
the calculable capacitor



Cross capacitor with movable guard electrode. 1, 2, 3, and 4 are the four cylindrical electrodes to which the cross-capacitor theorem is applies. 5 and 6 are the two guard electrodes; electrode 6 can be moved axially between two positions; the motion is monitored by a laser interferometer 7.

$C = \epsilon_0 \dfrac{\log 2}{\pi} \ell$, where ℓ is a geometrical length to be measured.

The calculable capacitor



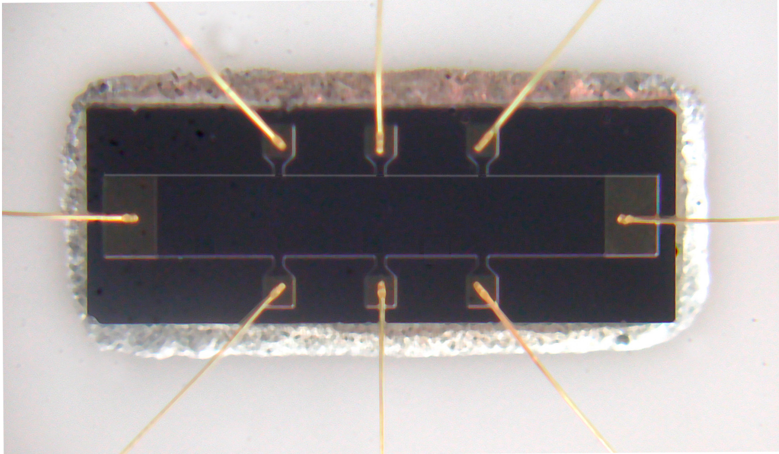
2015: NMIA-BIPM cross capacitor, with movable guard. (courtesy of J. Fiander)

Quantum electrical metrology experiments

Macroscopic quantum effect that display an electrical quantity related to fundamental constants

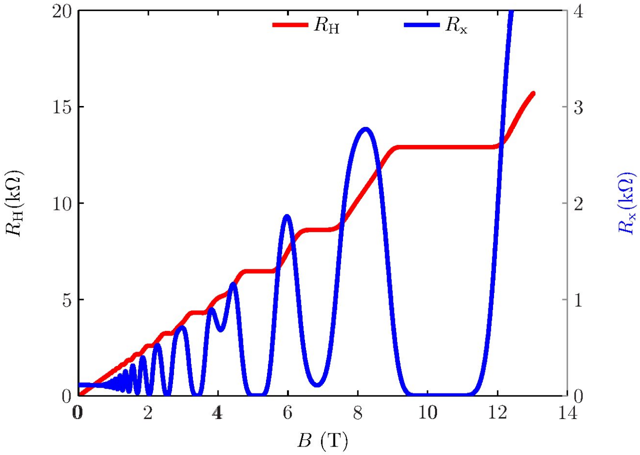
- quantized resistance: the quantum Hall effect
- quantized flux counting: the Josephson effect
- quantized charge counting: single-electron counting devices

The quantum Hall effect



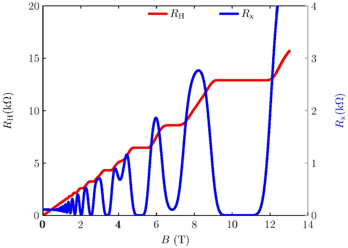
AlGaAs/GaAs Hall bar heterostructure, 1 mm × 0.4 mm;

The quantum Hall effect



- 🟥 $R_H = V_H/I$ Hall resistance;
- 🟦 $R_x = V_x/I$ longitudinal resistance.

The quantum Hall effect



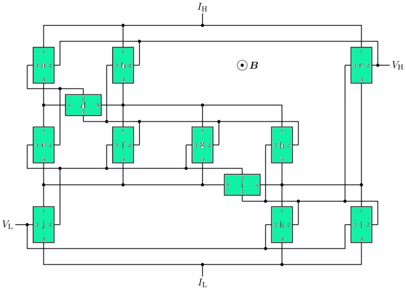
Each plateau i is centered has a resistance value $R_H = R_K/i$, with i integer

$$R_K = \frac{h}{e^2} = \frac{\mu_0\, c}{2\alpha}.$$

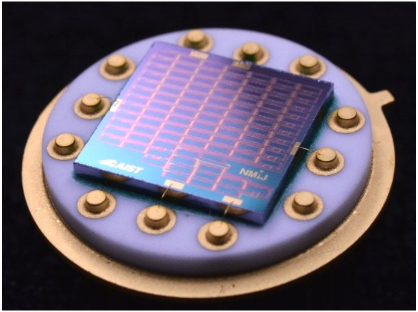
R_K is linked to the fine structure constant α which can be measured by non-electrical means.

CODATA least-squares adjustment: $R_K = 25\,812.807\,443\,4(84)\,\Omega \quad [3.2 \times 10^{-10}]$.

Quantum Hall array resistance standards



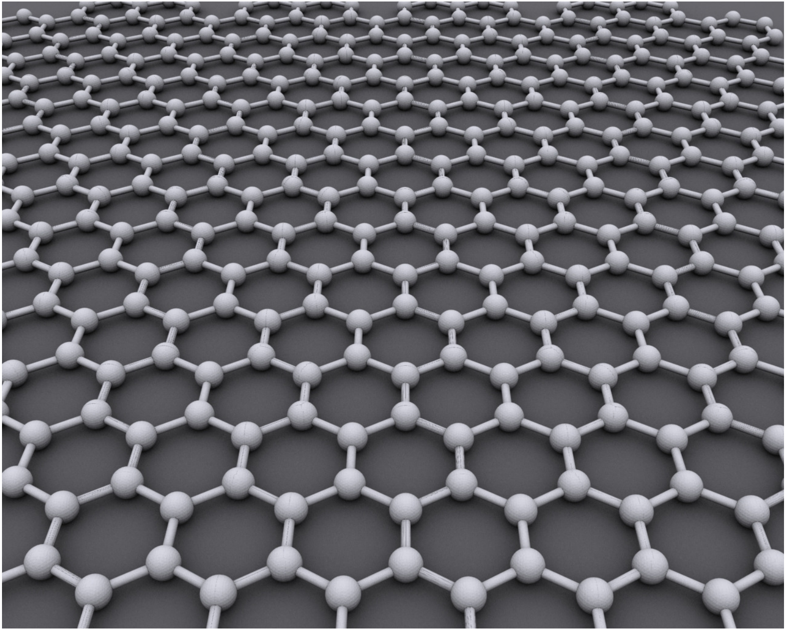
(a) 10 kΩ QHARS design (Ortolano et al., 2015)



(b) 1 MΩ QHARS (Oe et al., 2016)

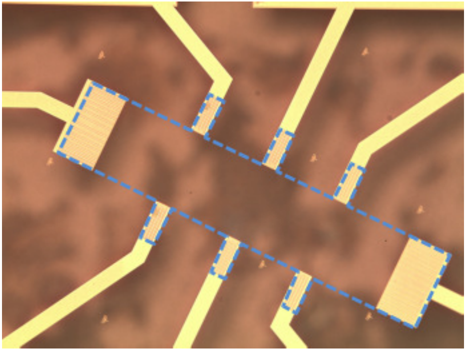
10 kΩ array: $R_{10\,\mathrm{k}\Omega} = \frac{203}{262} R_\mathrm{H} = (1 - 3.4 \times 10^{-8}) \times 10\,\mathrm{k}\Omega$

Graphene for QHE



Graphene for QHE

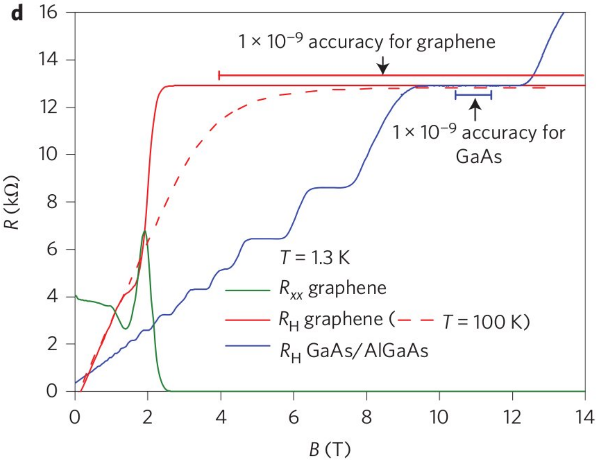
PTB graphene Hall bar



PTB

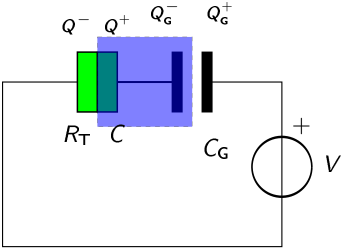
Graphene for QHE

(Ribeiro-Palau et al., 2015)

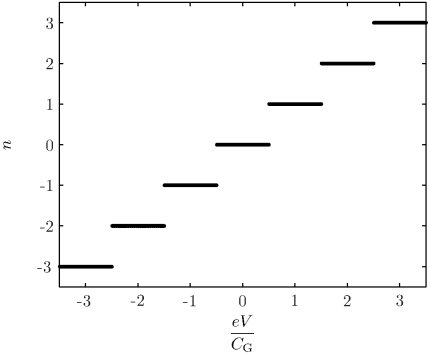


Quantized charge counting

Single charge confinement



Single-electron box, coupled to an external circuit with a tunnel junction (with tunnel resistance R_T and capacitance C) and a capacitor C_G.



occupation number n versus applied bias voltage V.

Quantized charge counting

Moving individual electrons

Via tunneling events, electrons charge the island with charge $Q_{\mathrm{i}} = -n\,e$, where n is an integer and e the charge quantum. The gate has capacitance C_{G} and holds charge Q_{G}; the tunnel junction has tunnel resistance R_{T}, capacitance C and holds charge Q; then, $Q_{\mathrm{i}} = Q - Q_{\mathrm{G}}$.

Circuit analysis of the mesh of Fig. **??** gives

$$E = \frac{Q^2}{2C} + \frac{Q_{\mathrm{G}}^2}{2C_{\mathrm{G}}} = \frac{C\,C_{\mathrm{G}}\,V^2 + Q_{\mathrm{i}}^2}{2(C + C_{\mathrm{G}})},$$

the generator work $W = Q_{\mathrm{G}}\,V$, and the free energy F

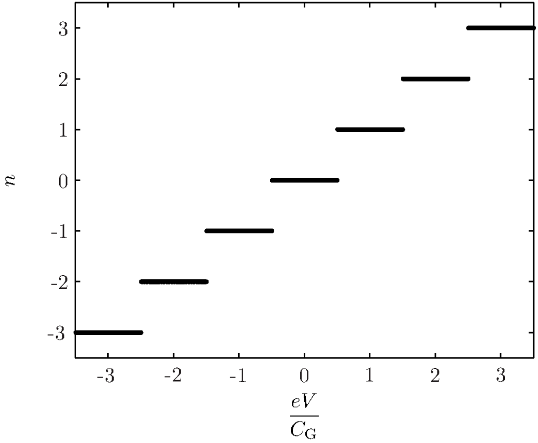
$$F = E - W = \frac{(C_{\mathrm{G}}\,V + Q_{\mathrm{i}})^2}{C + C_{\mathrm{G}}} + K = \frac{(C_{\mathrm{G}}\,V - n\,e)^2}{C + C_{\mathrm{G}}} + K$$

can be computed (K is a constant term).

Quantized charge counting

Moving individual electrons

At equilibrium at a given bias V, the minimization of the free energy $F(V)$ gives the corresponding equilibrium electron occupation of the box $n(V)$



Single-electron box occupation number n versus applied bias voltage V.

Quantized charge counting

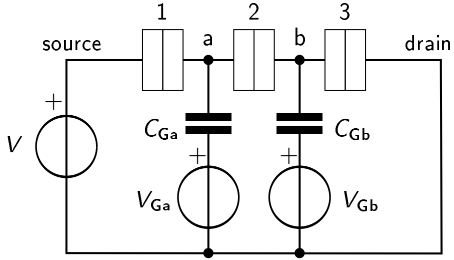
Conditions

In the derivation above, two hypoteses have been made:

1. the spacing between energy levels of the single electron box is large with respect to the average thermal excitation: $\frac{e^2}{2(C + C_G)} \gg k_B\Theta$. With nanofabrication techniques, device capacitances in the fF range can be achieved; an adequate working temperature lies in the tens of mK range.
2. the spacing between energy levels of the single electron box is large with respect to the energy uncertainty of an occupation state, in turn related by uncertainty principle to the state lifetime $R_T C$ caused by tunneling events. This gives the condition $R_T \gg R_K$.

Quantized charge counting

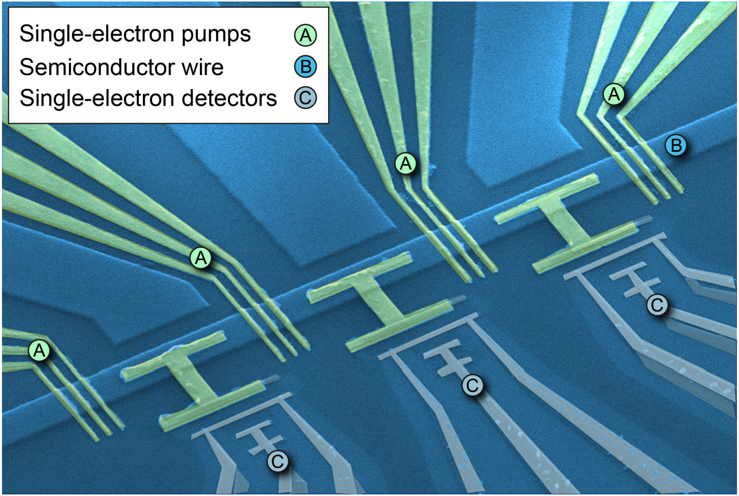
Nanodevices



A three-junction single-electron pump.

Quantized charge counting

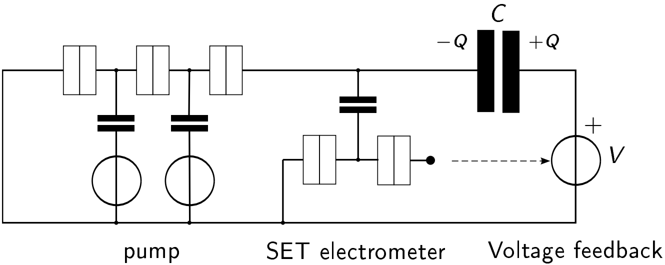
Nanodevices



Semiconductor single-electron pumps (courtesy PTB).

The Electron-counting capacitance standard

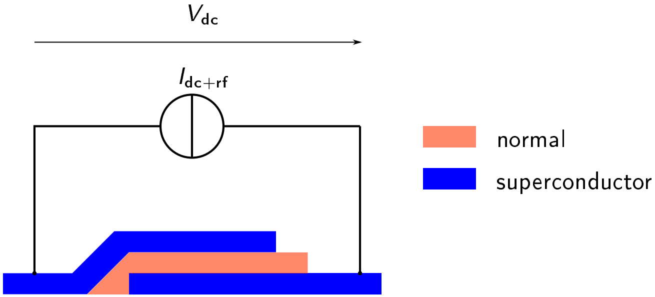
The definition of capacitance $Q = C\,V$ is directly employed in dc regime.



Electron-counting capacitance standards block schematics, using single-electron devices. A single-electron pump charges capacitor C to Q; a SET electrometer nulls low voltage side of C by driving a feedback generator to voltage V.

Counting flux quanta

Josephson junctions



Josephson junction:

- two superconductors coupled by a tunneling barrier
- have coupled wavefunctions

Counting flux quanta

The Josephson effect

$$i(t) = I_\mathrm{c} \sin\left(2\pi \frac{\phi(t)}{\Phi_0}\right)$$

where

- $\Phi_0 = 2e/h = 2.067\,833\,831(13) \times 10^{-15}\,\mathrm{Wb} \quad [6.1 \times 10^{-9}]$ (CODATA 2014) is the flux quantum;
- $K_\mathrm{J} = h/2e = \Phi_0^{-1} = 483\,597.8525(30)\,\mathrm{GHz/V}$ is the Josephson constant;
- I_c is the critical current of the junction;
- $\phi(t) = \displaystyle\int_0^t v(\tau)\,\mathrm{d}\tau$ is the flux of the voltage applied to the junction.

Counting flux quanta

voltage to frequency converter: the AC Josephson effect

Applying a constant voltage V to the junction, $\phi(t) = Vt$,

$$i(t) = I_c \cos\left(\frac{2\pi}{\Phi_0} Vt\right)$$

which is an oscillator with frequency $f = \dfrac{V}{\Phi_0}$

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

Applying a dc+ac voltage excitation $v(t) = V_{\mathrm{dc}} + V_{\mathrm{ac}} \cos(2\pi f_{\mathrm{ac}} t)$, the Josephson carrier $f_{\mathrm{J}} = V_{\mathrm{dc}}/\Phi_0$ is FM modulated.
The FM sidebands allow a zero-frequency (dc) current bias only for the condition $f_{\mathrm{J}} = n f_{\mathrm{ac}}$, integer n:

$$V_{\mathrm{dc}} = n\Phi_0 f_{\mathrm{ac}} = \frac{n f_{\mathrm{ac}}}{K_{\mathrm{J}}}$$

Every cycle of f_{ac}, n flux quanta are counted across the junction.
Feasible drive frequencies: $f_{\mathrm{ac}} = 70\,\mathrm{GHz} \quad \Rightarrow \quad V_{\mathrm{dc}} = 150\,\mu\mathrm{V}$.

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

Under proper I_{rf} excitation amplitude of frequency f_{rf}

$$V_{\mathrm{dc}} = n\Phi_0 f_{\mathrm{rf}} = \frac{n}{K_{\mathrm{J}}} f_{\mathrm{rf}}$$

where

$\Phi_0 = h/2e = 2.067\,833\,831(13) \times 10^{-15}\,\mathrm{Wb} \quad [6.1 \times 10^{-9}]$ is the flux quantum;

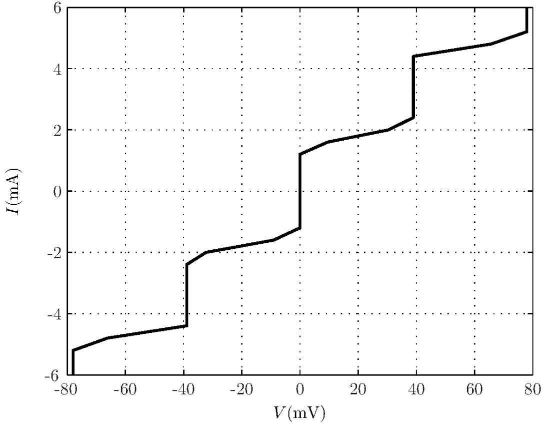
$K_{\mathrm{J}} = 2e/h = 1/\Phi_0 = 483\,597.8525(30)\,\mathrm{GHz/V}$ is the Josephson constant;

n is a small integer.

Feasible drive frequencies:
$f_{\mathrm{rf}} = 70\,\mathrm{GHz} \quad \Rightarrow \quad V_{\mathrm{dc}} = 150\,\mu\mathrm{V}$.

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

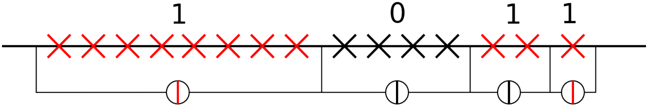


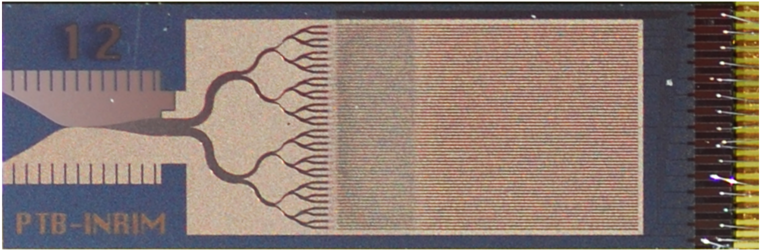
The $I - V$ characteristic of a Josephson array (256 junctions) under microwave irradiation. Steps $n = 0, \pm 1, \pm 2$ are visible. $f \approx 73\,\mathrm{GHz}$

Counting flux quanta

Josephson binary DAC

Binary-weighted Josephson DAC





Josephson junction binary array chip. 13 bit+sign DAC with 8192 superconducting-normal metal-insulator-superconductor (SNIS) junctions. The junctions are geometrically arranged over 32 parallel strips of 256 junctions each. $f = 70\,\mathrm{GHz}$. $V_{\mathrm{fullscale}} \approx \pm 1.2\,\mathrm{V}$

The quantum experiments in the framework of the present SI

Knowledge in 1989 (CODATA):

$K_\mathrm{J} = 483\,597.9(2)\,\mathrm{GHz/V} \qquad [4 \times 10^{-7}]$

$R_\mathrm{K} = 25\,812.807(5)\,\Omega \qquad [2 \times 10^{-7}]$

but, *reproducibility* of Josephson and quantum Hall experiments in different experiments and different laboratories was much higher: 10^{-9}–10^{-10}

Solution: invent non-SI units! 18th CGPM resolution 6: Valid since January 1, 1990:

$K_\mathrm{J\text{-}90} = 483\,597.9\,\mathrm{GHz/V} \qquad [\mathrm{exact}]$

$R_\mathrm{K\text{-}90} = 25\,812.807\,\Omega \qquad [\mathrm{exact}]$

To $K_\mathrm{J\text{-}90}$ and $R_\mathrm{K\text{-}90}$ the conventional units Ω_{90}, H_{90}, F_{90}, A_{90}, W_{90} are associated.
These are the electrical units in use nowadays.

The quantum experiments in the present SI

Present status of the conventional units

Becuase of improvements in the measurement of fundamental constants, today (CODATA 2014)

$$K_\mathrm{J} = 483\,597.8525(30)\,\mathrm{GHz/V} \qquad [6.1 \times 10^{-9}]$$

$$R_\mathrm{K} = 25\,812.807\,455\,5(59)\,\Omega \qquad [2.3 \times 10^{-10}]$$

Therefore

$$V_{90} = 1+9.8(6) \times 10^{-8}\,\mathrm{V}$$

$$\Omega_{90} = 1-1.764(2) \times 10^{-8}\,\Omega$$

$\Rightarrow$ Unacceptable deviation of the conventional units respect to the SI units

The forthcoming SI

Redefinition of the SI base of interest for electromagnetism:

- kg the kilogram;
- A the ampere;

by fixing the values of the fundamental constants:

- h Planck constant;
- e elementary charge;

The forthcoming SI: the base unit ampere

The ampere will be redefined as:

> *The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\,176\,621 \times 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{\mathrm{Cs}}$.*

The kilogram will be redefined as:

> *The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.626\,070\,040 \times 10^{-34}$ when expressed in the unit J s, which is equal to $\mathrm{kg\,m^2\,s^{-1}}$, where the metre and the second are defined in terms of c and $\Delta\nu_{\mathrm{Cs}}$.*

The forthcoming SI: realization of the units

Consequences of the redefinition

e will be **exact**;

$\Rightarrow$ any electron-counting experiment will be a realization of the ampere;

$R_K = \dfrac{h}{e^2}$ will be **exact**;

$\Rightarrow$ the quantum Hall effect will be a realization of the ohm;

$K_J = \dfrac{2e}{h}$ will be **exact**;

$\Rightarrow$ the Josephson effect will be a realization of the volt;

$\Rightarrow$ Combining Josephson and quantum Hall effects with Ohm's law will be a realization of the ampere.

The forthcoming SI: electromagnetic fundamental constants

μ_0 the magnetic constant will be no more $4\pi \times 10^{-7}$ H/m: not exact and subject of measurement;

$\epsilon_0 = \dfrac{1}{\mu_0 c^2}$ the electric constant will be no more exact;

$\Rightarrow$ ϵ_0 and μ_0 will have the same relative uncertainty and will be totally correlated (correlation coefficient $= -1$)

$Z_0 = \mu_0 c$ the impedance of free space, and

$Y_0 = (\mu_0 c)^{-1}$ the admittance of free space will be no more exact;

The forthcoming SI: electromagnetic fundamental constants

The fine-structure constant

$$\alpha = \frac{e^2}{\epsilon_0 hc}$$

$$\alpha^{-1} = 2\frac{R_K}{Z_0} = 137.035\,999\,139(31)$$

is not exact, but can be measured with very high accuracy (2.3×10^{-10} CODATA 2014) via atomic spectroscopy experiments.

The forthcoming SI: realization of the units

A new role for the mechanical experiments

h will be **exact**;

$\Rightarrow$ The Kibble balance, if traceable to K_J and R_K, will be a realization of the kilogram.

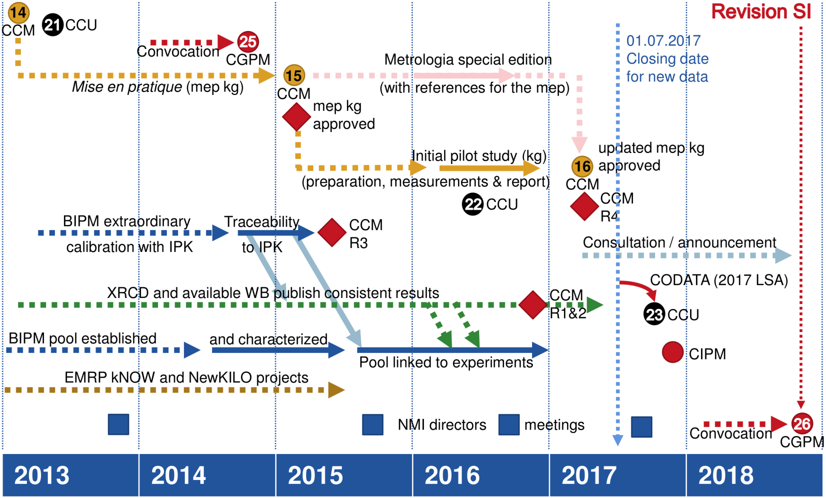
(and similarly for the voltage and the current balances)

μ_0 has the same uncertainty of α (2.3×10^{-10}),

$\Rightarrow$ the calculable inductor keeps the status of a practical realization of the henry;

$\Rightarrow$ the calculable capacitor keeps the status of a practical realization of the farad.

Joint CCM and CCU roadmap for the new SI



Revision SI

14 CCM

21 CCU

Convocation

25 CGPM

Mise en pratique (mep kg)

15 CCM

Metrologia special edition

(with references for the mep)

mep kg approved

01.07.2017

Closing date for new data

Initial pilot study (kg)

(preparation, measurements & report)

updated mep kg approved

16 CCM

22 CCU

CCM R4

BIPM extraordinary calibration with IPK

Traceability to IPK

CCM R3

Consultation / announcement

CODATA (2017 LSA)

XRCD and available WB publish consistent results

CCM R1&2

23 CCU

BIPM pool established

and characterized

Pool linked to experiments

CIPM

EMRP kNOW and NewKILO projects

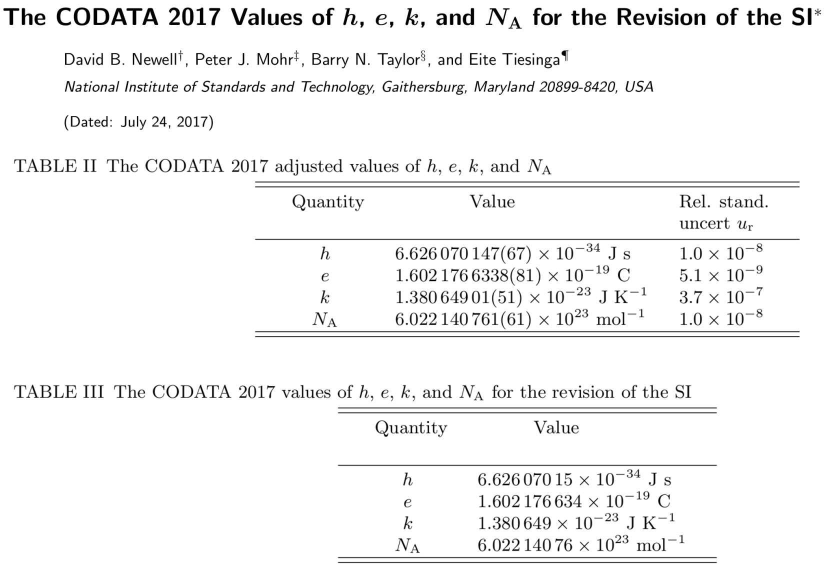
NMI directors meetings

Convocation

26 CGPM

2013 2014 2015 2016 2017 2018

Breaking news: CODATA 2017 adjustment!



The CODATA 2017 Values of h, e, k, and N_A for the Revision of the SI*

David B. Newell†, Peter J. Mohr‡, Barry N. Taylor§, and Eite Tiesinga¶

National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8420, USA

(Dated: July 24, 2017)

TABLE II The CODATA 2017 adjusted values of h, e, k, and N_A

Quantity	Value	Rel. stand. uncert u_r
h	$6.626\,070\,147(67) \times 10^{-34}$ J s	1.0×10^{-8}
e	$1.602\,176\,6338(81) \times 10^{-19}$ C	5.1×10^{-9}
k	$1.380\,649\,01(51) \times 10^{-23}$ J K^{-1}	3.7×10^{-7}
N_A	$6.022\,140\,761(61) \times 10^{23}$ mol^{-1}	1.0×10^{-8}

TABLE III The CODATA 2017 values of h, e, k, and N_A for the revision of the SI

Quantity	Value
h	$6.626\,070\,15 \times 10^{-34}$ J s
e	$1.602\,176\,634 \times 10^{-19}$ C
k	$1.380\,649 \times 10^{-23}$ J K^{-1}
N_A	$6.022\,140\,76 \times 10^{23}$ mol^{-1}

2017.08.27: stil unpublished

Breaking news: CODATA 2017 adjustment!

Quantity	Value	Re. std	year	change
h	$6.626\,070\,040(81) \times 10^{-34}$ J s	1.2×10^{-8}	2014	
h	$6.626\,070\,147(67) \times 10^{-34}$ J s	1.0×10^{-8}	2017	1.6×10^{-8}
e	$1.602\,176\,620\,8(98) \times 10^{-19}$ C	6.1×10^{-9}	2014	
e	$1.602\,176\,633\,8(81) \times 10^{-19}$ C	5.1×10^{-9}	2017	8.1×10^{-9}

New SI Implementation day?

May 20, 2019

World Metrology Day

Yet to be confirmed, but stay prepared!

A short video summary of this lecture:


QUANTUM METROLOGY FOR THE PRACTICAL REALIZATION OF ELECTRICAL UNITS, IN THE FRAMEWORK OF THE NEW SI

Luca Callegaro

https://ieeetv.ieee.org/ieeetv-specials/quantum-metrology-for-the-practical-realization-of-electrical-units-in-the-framework-of-the-new-si?

Further reading

- "Draft of the 9th SI brochure," 10 Nov 2016
- CCEM Working Group on the SI, "Mise en pratique for the ampere and other electric units in the international system of units," 2017, CCEM-17-08
- P. J. Mohr, D. B. Newell, and B. N. Taylor, "CODATA recommended values of the fundamental physical constants: 2014," *J. Phys. Chem. Ref. Data*, vol. 45, 2016
- J. Fischer and J. Ullrich, "The new system of units," *Nature Physics*, vol. 12, pp. 4–7, 2016
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