Redefining the measurement units

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The need for measurement units
Legal metrology

Do not have in your bag different weights, a great and a small; Or in your house different measures, a great and a small. But have a true weight and a true measure . . .

Non avrai nel tuo sacchetto due pesi diversi, uno grande e uno piccolo. Non avrai in casa due tipi di misure, una grande e una piccola. Terrai un peso completo e giusto, terrai una misura completa e giusta . . .

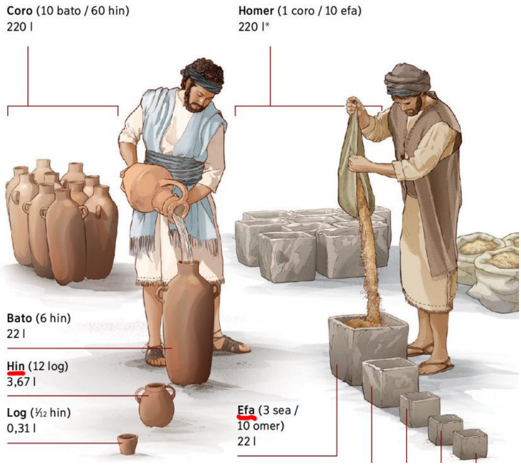
[Deuteronomy 25:13-16]

The need for measurement units

Use honest scales and honest weights, an honest ephah and an honest hin.

Avrete stadere giuste, pesi giusti, efa giusto, hin giusto.

[Leviticus 19:36]



One system of units as foundation for the nation

Una mensura vini sit per totum regnum nostrum, et una mensura cervisie, et una mensura bladi, scilicet quarterium Londoniense, et una latitudo pannorum tinctorum et russetorum et halbergettorum, scilicet due ulne infra listas; de ponderibus autem sit ut de mensuris ...

[Magna Charta Libertatum (1215), Clause 35]

1 London quarter = 225 l

The French revolution

Cleaning up the mess

Longueur de quelques pieds de France.

		ligne.	centre.
Besançon, le pied est de		137	10
Dijon.		139	20
Dôle.		150	30
Grenoble.		151	20
Lorraine.		127	«
Lyon.		151	50
Mâcon.		148	20
Paris.		144	«
Rouen.		120	«
Sedan.		123	«
Strasbourg.	Pied de ville.	128	27
	Pied de campagne.	130	90
Vienne en Dauphiné.		143	«

Mesures pour les Aunages.

Aunes.	Abbeville, l'aune est de		524	
	Arras.		309	40
	Bayonne.		391	80
	Bordeaux.		528	«
	Bretagne.		597	20
	Caen.		524	«
	Cambrai.		317	60
	Dunkerque.		299	80
	Lille.		305	60
	Lyon.		520	50
	Paris.	Pour les Soieries.	527	50
		Pour les Lainages.	526	40
		Pour les Toileries.	524	«
	Morlaix.		597	20
	Nantes.		526	«
	Rouen.	Pour les Lainages.	516	«
		Pour les Toileries.	519	20
	Saint-Malo.		597	20

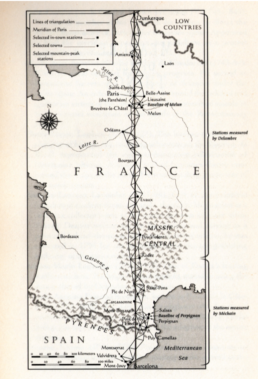
Suite des Mesures pour les Aunages.

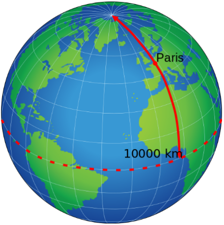
Canne.	Toulon.	859	60
	Montpellier.	891	60
	Provence.	888	90
	Toulouse.	807	«
	Troyes.	351	70

Mesures rondes pour les choses sèches.

		Pouces cubes
Anées.	Lyon.	9670
	Mâcon.	12893
Bichets.	Châlons-sur-Saône.	9283
	Verdun.	9670
Boisseaux.	Amboise.	552
	Aurai.	1934
	Blois.	387
	Bordeaux.	3868
	Bourbon-Lancy.	573
	La Charité.	967
	Charolles.	1221
	Châteauneuf-sous-Loire.	1105
	Cosne.	314
	Dieppe.	5157
	Havre-de-Grace.	1743
	Honfleur.	1976
	Montreuil.	430
	Morlaix.	2670
	Nevers.	967
	Paris.	644⅔
	Périgueux.	1547
	Roanne.	967
	La Rochelle.	1658
	Rouen.	128
	Tours.	542
	Villeneuve d'Agénois.	4100
	Briare.	703
	Cahors.	1469

The French revolution: the metric system





$1\,\mathrm{s}$: $\dfrac{1}{24 \times 60 \times 60}$ of the mean solar day
(Earth's rotation period);

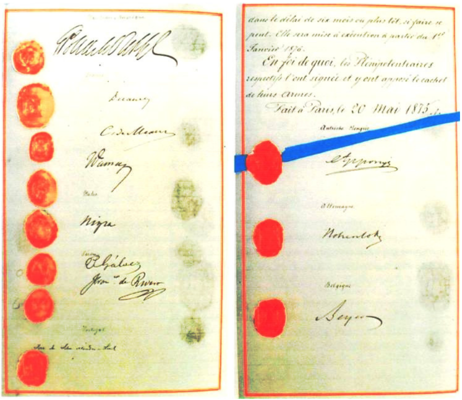
$1\,\mathrm{m}$: $\dfrac{1}{10\,000\,000}$ of $\dfrac{1}{4}$ of Earth's meridian
(from pole to equator);

$1\,\mathrm{kg}$: $1\,\mathrm{dm}^3$ of water at its maximum density

Universal units: for everybody, for all times

The Metre Convention

Paris, 20 May 1875: an international treaty

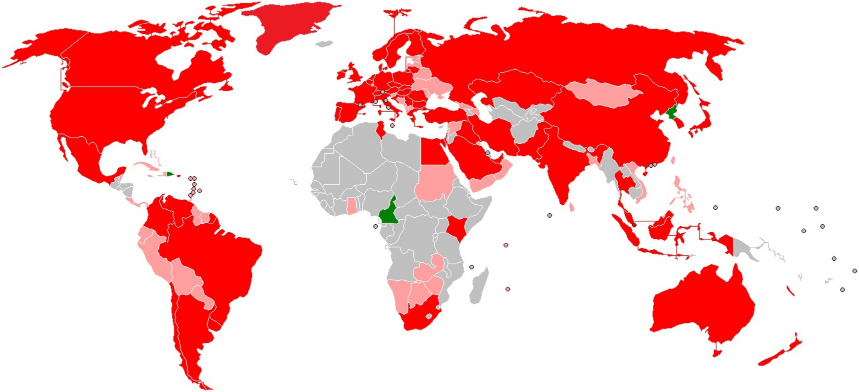


Original signatories: Argentina, Austria-Hungary, Belgium, Brazil, Denmark, France, Germany, Italy, Peru, Portugal, Russia, Spain, Sweden and Norway, Switzerland, Turkey, United States of America, and Venezuela

[for His Majesty the King of Italy: Chevalier Constantino Nigra, Knight of the Grand Cross of his Orders of St. Maurice and St. Lazarus, and of the Crown of Italy, Grand Officer of the Legion of Honor, . . . Extraordinary and Minister Plenipotentiary at Paris]

The Metre Convention

The signatories today



The present International System of units (SI)

The seven base units

- m — The **metre** is the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second.
- kg — The **kilogram** is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
- s — The **second** is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.
- A — The **ampere** is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
- K — The **kelvin**, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
- mol — The **mole** is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12.
- cd — The **candela** is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

Base and derived units

Base units

Symbol	Unit name
s	second
m	metre
kg	kilogram
A	ampere
K	kelvin
mol	mole
cd	candela

Base and derived units

Base units

Symbol	Unit name
s	second
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mol	mole
cd	candela

Derived units

$$\mathrm{s}^{\alpha}\ \mathrm{m}^{\beta}\ \mathrm{kg}^{\gamma}\ \mathrm{A}^{\delta}\ \mathrm{K}^{\epsilon}\ \mathrm{mol}^{\zeta}\ \mathrm{cd}^{\eta},$$

where α, β, γ, δ, ϵ, ζ and η are (usually) integers.

The International System of units (SI

many derived units

Examples

m/s unit of velocity;

$W = kg\,m/s^2$ unit of power;

$V = kgm^2s^{-3}A^{-1}$ unit of electrical potential difference (voltage)

SI units for electromagnetic quantities

Derived units with special names

Derived quantity	name	symbol	expression in terms of base units
frequency	hertz	Hz	s^{-1}
energy	joule	J	$m^2\,kg\,s^{-2}$
power	watt	W	$m^2\,kg\,s^{-3}$
electric charge	coulomb	C	$s\,A$
electric potential difference	volt	V	$m^2\,kg\,s^{-3}\,A^{-1}$
electric capacitance	farad	F	$m^{-2}\,kg^{-1}\,s^{-4}\,A^2$
electric resistance	ohm	Ω	$m^2\,kg\,s^{-3}\,A^{-2}$
electric conductance	siemens	S	$m^{-2}\,kg^{-1}\,s^3\,A^2$
magnetic flux	weber	Wb	$m^2\,kg\,s^{-2}\,A^{-1}$
magnetic flux density	tesla	T	$kg\,s^{-2}\,A^{-1}$
inductance	henry	H	$m^2\,kg\,s^{-2}\,A^{-2}$

SI prefixes and suffixes

The SI adopts a series of prefix names and prefix symbols to form the names and symbols of the decimal multiples and submultiples of units, ranging from 10^{24} to 10^{-24}.

name	symbol	factor	name	symbol	factor
yocto	y	10^{-24}	deca	da	10^{1}
zepto	z	10^{-21}	hecto	h	10^{2}
atto	a	10^{-18}	kilo	k	10^{3}
femto	f	10^{-15}	mega	M	10^{6}
pico	p	10^{-12}	giga	G	10^{9}
nano	n	10^{-9}	tera	T	10^{12}
micro	μ	10^{-6}	peta	P	10^{15}
milli	m	10^{-3}	exa	E	10^{18}
centi	c	10^{-2}	zetta	Z	10^{21}
deci	d	10^{-1}	yotta	Y	10^{24}

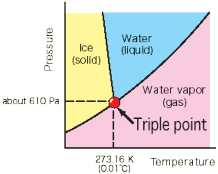
The expression of the value of electromagnetic quantities benefits of large or small prefixes, more often than in other scientific fields. For example, it is common to speak of fA current, PΩ resistance, or aF capacitance values.

Definition of units

in the present SI

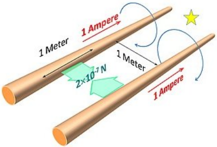
an **artefact**:

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.



a **natural property**

The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.



an **idealized experiment**

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length [. . .] would produce a force equal to 2×10^{-7} *newton per metre of length*

The realization of the units

Realization (VIM 5.1 ↗)

The realization of the definition of a unit can be provided by a measuring system, a material measure, or a reference material.

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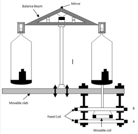
Examples

an artefact:

The international prototype of the kilogram is the realization of the kilogram.

a device

A triple point of water cell is a realization of the kelvin.



an experiment

The current balance is a realization of the ampere.

Defining and realizing units

Reproduction (VIM 5.1 ↗)

The *reproduction* of a unit consists in realizing the unit not from its definition but in setting up a highly reproducible measurement standard based on a physical phenomenon, and, usually, by assigning to it a conventional value.

Defining and realizing units

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Examples

In the present SI:

- The volt is reproduced by means of the Josephson effect.
- The ohm is reproduced by means of the quantum Hall effect.
- The thermodynamic temperature scale is reproduced through two conventional temperature scales, the *International Temperature Scale of 1990* (ITS-90) and the *Provisional Low Temperature Scale of 2000* (PLTS-2000).

The ampere

In the present SI, the definition of the base unit ampere is mechanical:

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.*

All electromagnetic derived units have an ultimately mechanical definition also.

These quantities are **exact**:

$\mu_0 = 4\pi \times 10^{-7}$ H/m the *magnetic constant*;

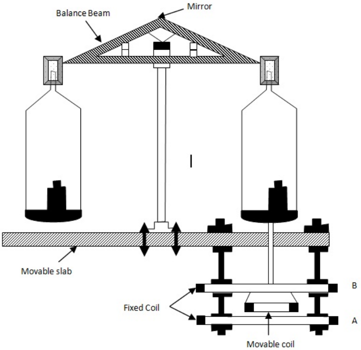
$\epsilon_0 = \left(\mu_0 c^2\right)^{-1} = 8.854\,187\,817\ldots$ pF/m, the *electric constant*

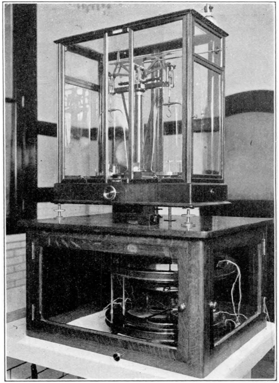
$Z_0 = \mu_0\, c = \sqrt{\mu_0\, \epsilon_0^{-1}} = 376.730\,313\,4\ldots\ \Omega$, the *impedance of free space*

μ_0, ϵ_0 constant $\Rightarrow$ realization of SI units of impedance.

Realization of the ampere

The (electrodynamic) ampere balance (Vigoreux, 1965)





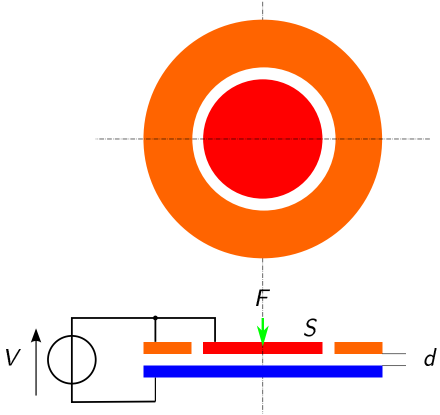
Ampère force law:

$$F = \frac{\mu_0}{4\pi} \int_{\Gamma_1} \int_{\Gamma_2} \frac{I_1 \, \mathrm{d}\boldsymbol{\ell}_1 \times I_2 \, \mathrm{d}\boldsymbol{\ell}_2 \times \boldsymbol{r}_{21}}{|\boldsymbol{r}_{21}|^2}$$

If $I_1 = I_2$, $F = \mu_0 k I^2$ where k is computed from geometrical measurements

Realization of the volt

The (electrostatic) voltage balance



Force between plates: $F = \epsilon_0 \frac{S}{2d^2} V^2 = \epsilon_0 \, k \, V^2$

where k is computed from geometrical measurements

Realization of the volt

Cylindrical-electrode voltage balance, PTB (Siencknecht and Funck, 1986)

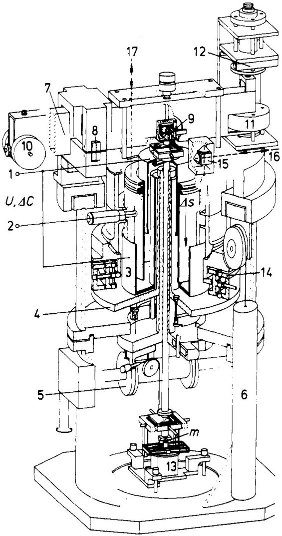
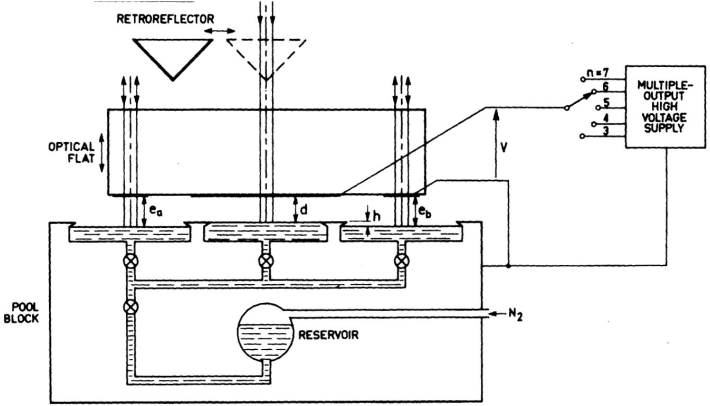


Fig. 1. Perspective view of the PTB voltage balance. 1 Inner electrode, 2 high-voltage electrode, 3 guard electrode, 4 carriage of displace unit, 5 driving device for displace unit, 6 counterweight of displace unit, 7 balance beam, 8 central joint of balance beam, 9 load joint of balance beam, 10 counterbalance weight, 11 position sensor, 12 retainer for balance beam, 13 load-changing device, 14 device for centering and vertical electrode adjustment, 15 interferometer for Δs-measurement, 16 light beam of interferometers for Δs-measurement, 17 light beam of autocollimator for vertical electrode adjustment

$V = 10\,186\,\mathrm{V} = 1000 \times E_{\mathrm{Weston}}$; $m = 2\,\mathrm{g}$!

Realization of the volt

Mercury-electrode elevation, CSIRO Australia (Sloggett et al., 1985)

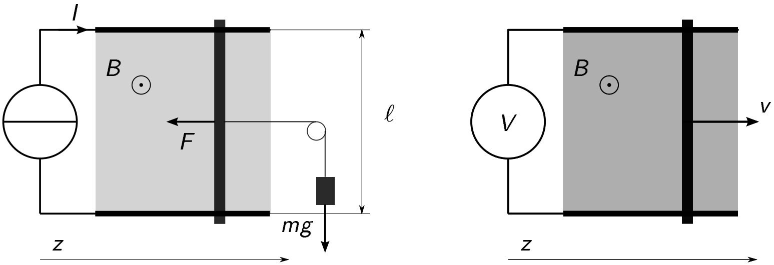


$$V = \sqrt{\frac{2\rho g}{\epsilon_0}} d\sqrt{h}. \; V = \text{kV}, \; d = 600\,\mu\text{m}, \; u_V = 0.33 \times 10^{-6}$$

Realization of the electrical watt

The watt balance, or Kibble balance

Solves the problem of geometrical measurements!



- Weighing mode: $F = B\ell I = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z} I$
- Moving mode: $E = \dfrac{\mathrm{d}\Phi}{\mathrm{d}t} = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z}\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z} v$
- $Fv = EI$; $P_{\mathrm{m}} = P_{\mathrm{e}}$

The Kibble balance
(Robinson and Schlamminger, 2016)

Solves the problem of geometrical measurements!

weighing mode

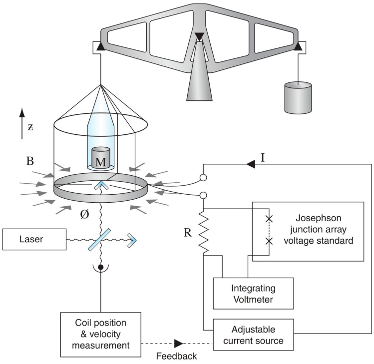


Figure 1. The Kibble balance in weighing mode.

moving mode

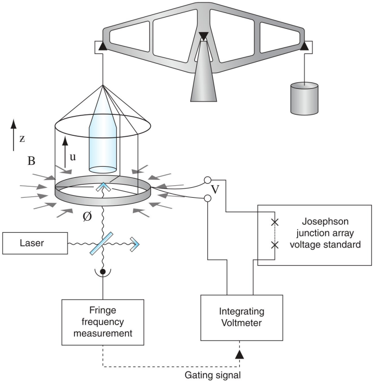
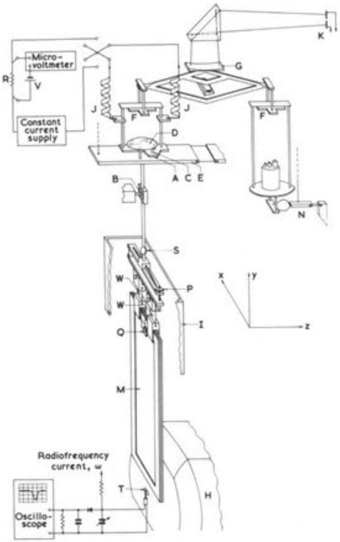


Figure 2. The Kibble balance in moving mode.

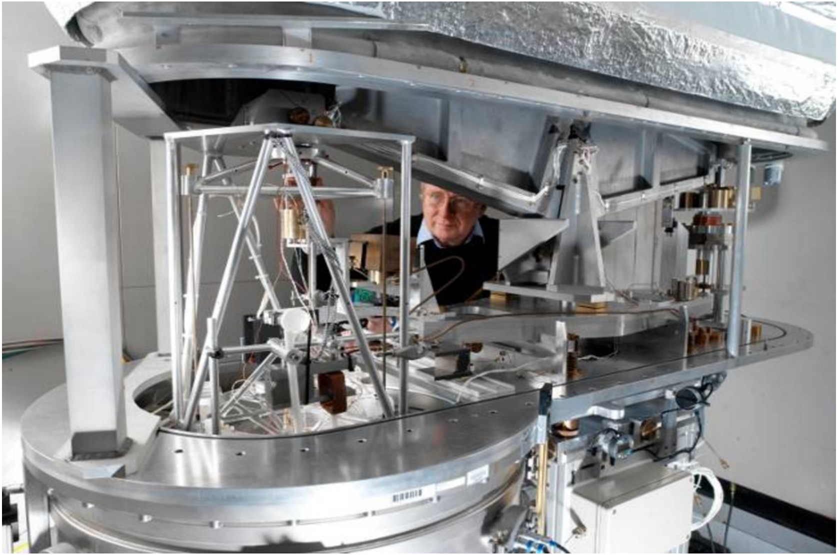
The Kibble balance evolution

NPL, Kibble (1976) for the gyromagnetic ratio of the proton



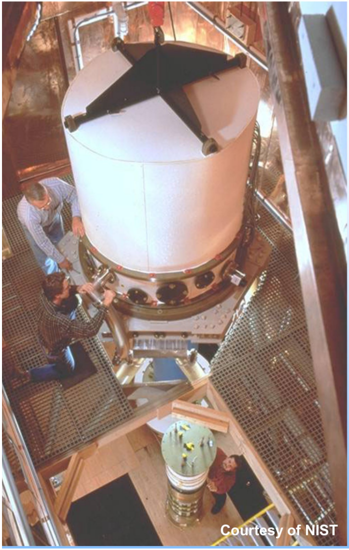
The Kibble balance: evolution

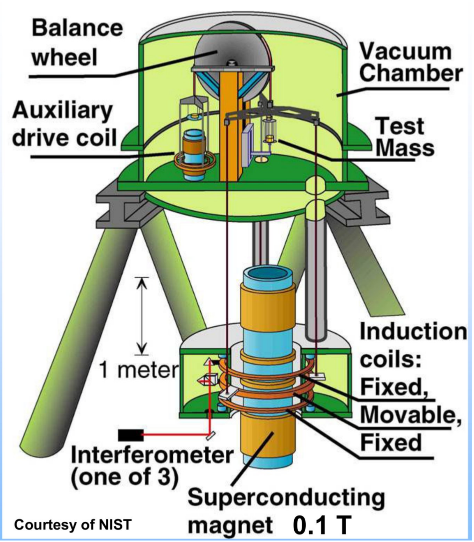
NRC, Bryan P. Kibble and I. Robinson, 2011



The Kibble balance: evolution

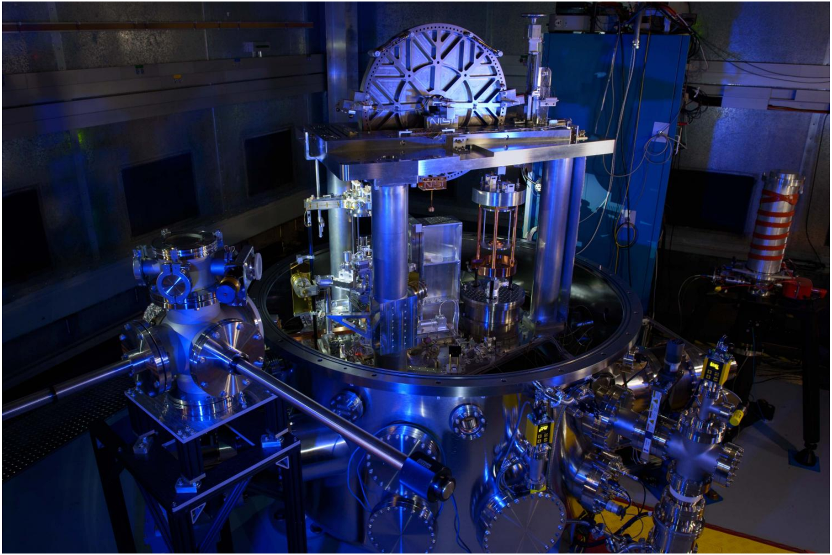
NIST-3





The Kibble balance: evolution

The next generation: NIST-4, 2016



The Kibble balance: evolution

The next generation: NPL, 2017

The Kibble balance

Determination of the Planck constant

To be discussed again after the quantum experiments

- $mgv = EI$
- $E = n\frac{f_\mathrm{E}}{K_\mathrm{J}}$
- $I = \frac{V_\mathrm{I}}{R} = \frac{f_\mathrm{I}}{K_\mathrm{J}}\frac{1}{rR_\mathrm{K}}$
- $K_\mathrm{J} = \frac{2e}{h}$
- $R_\mathrm{K} = \frac{h}{e^2}$

$$\Rightarrow mgv = hf_\mathrm{E}f_\mathrm{I}\frac{n}{r}$$

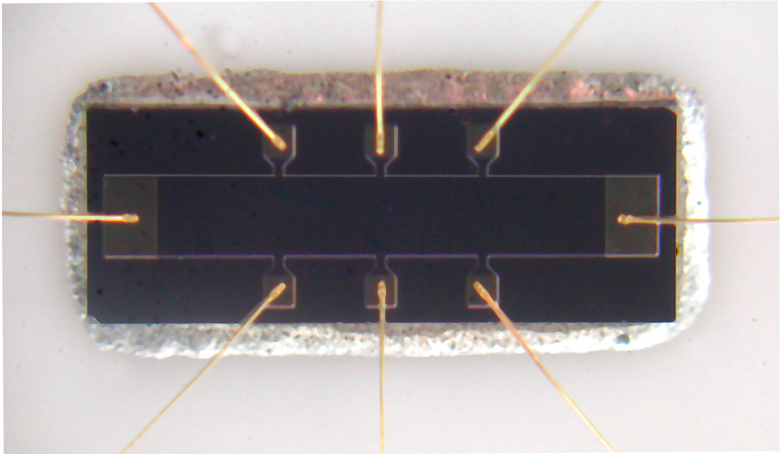
h can be measured mechanically

Quantum electrical metrology experiments

Macroscopic quantum effect that display an electrical quantity related to fundamental constants

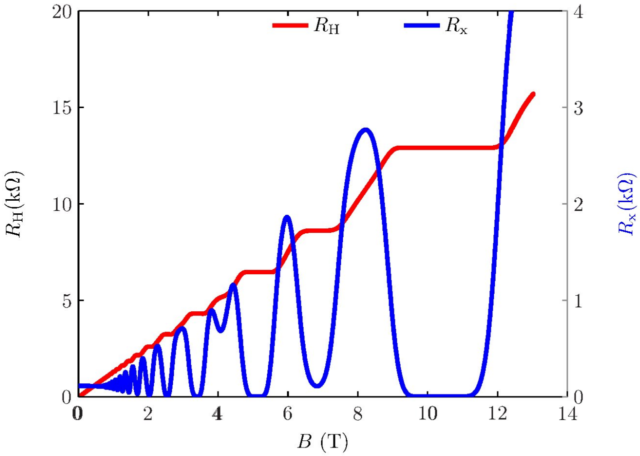
- quantized resistance: the quantum Hall effect
- quantized flux counting: the Josephson effect
- quantized charge counting: single-electron counting devices

The quantum Hall effect



AlGaAs/GaAs Hall bar heterostructure, 1 mm × 0.4 mm;

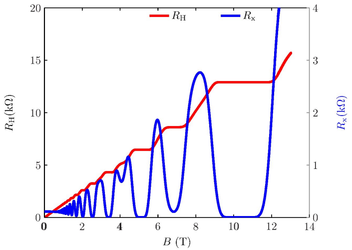
The quantum Hall effect



— $R_H = V_H/I$ Hall resistance;

— $R_x = V_x/I$ longitudinal resistance.

The quantum Hall effect

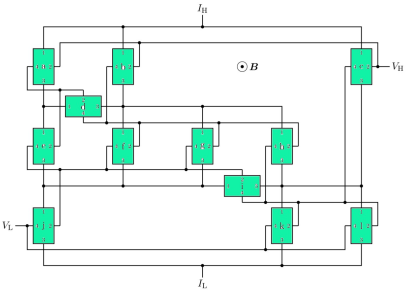


Each plateau i is centered on a resistance value $R_H = R_K/i$, with i integer

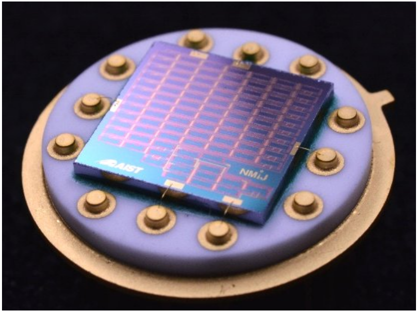
$$R_K = \frac{h}{e^2} = \frac{\mu_0\, c}{2\alpha}.$$

R_K is linked to the fine structure constant α which can be measured by non-electrical means.

Quantum Hall array resistance standards



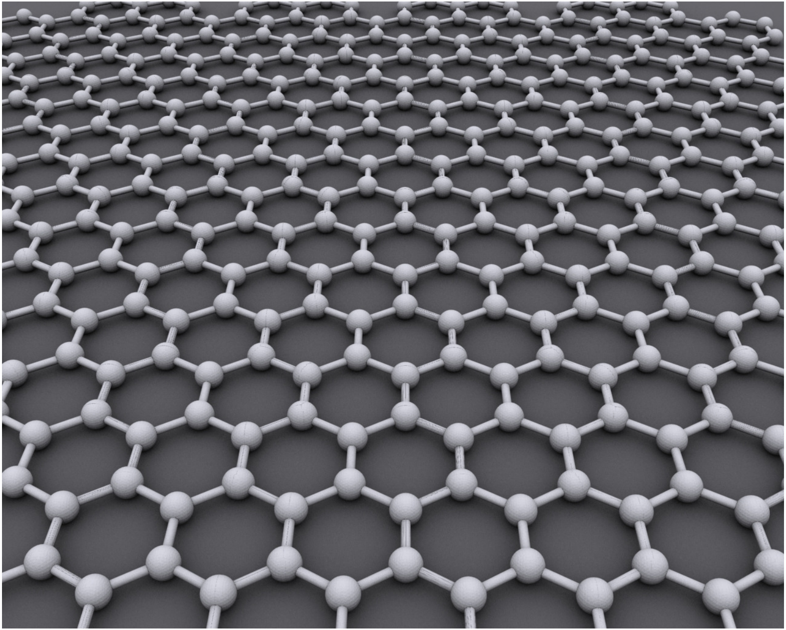
(a) 10 kΩ QHARS design (Ortolano et al., 2015)



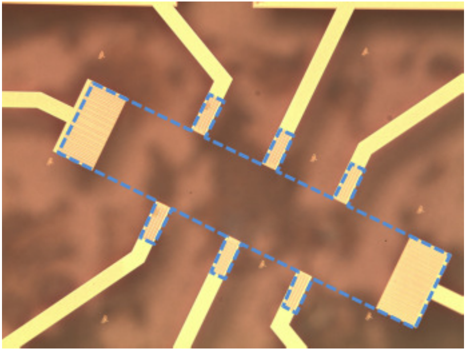
(b) 1 MΩ QHARS (Oe et al., 2016)

10 kΩ array: $R_{10\,k\Omega} = \frac{203}{262} R_H = (1 - 3.4 \times 10^{-8}) \times 10\,k\Omega$

Graphene for QHE

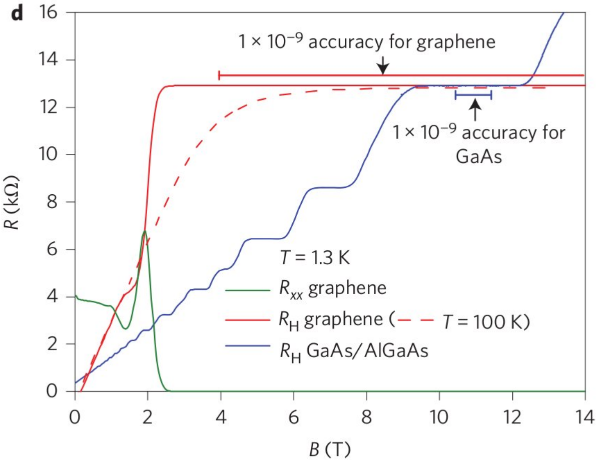


Graphene for QHE
PTB graphene Hall bar



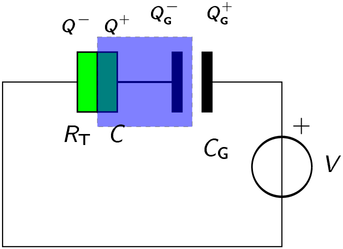
PTB

Graphene for QHE
(Ribeiro-Palau et al., 2015)

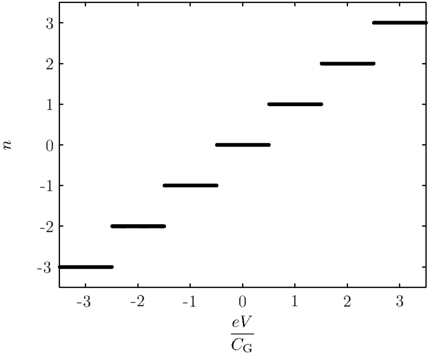


Quantized charge counting

Single charge confinement

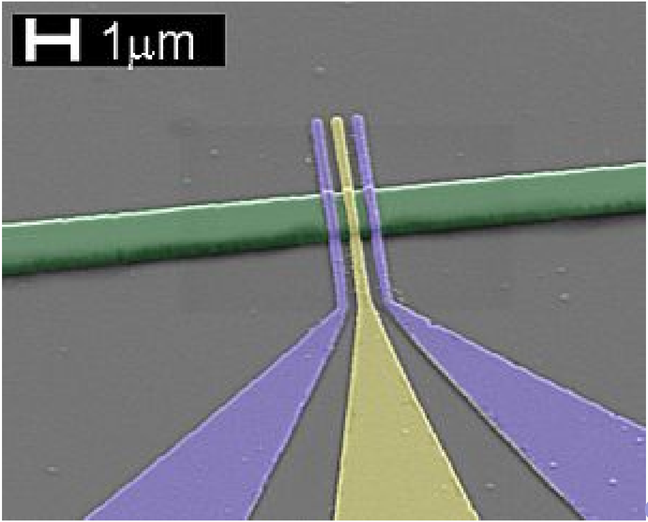


Single-electron box, coupled to an external circuit with a tunnel junction (with tunnel resistance R_T and capacitance C) and a capacitor C_G.



occupation number n versus applied bias voltage V.

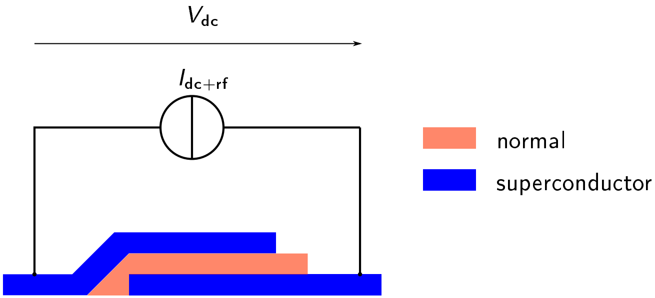
Quantized charge counting
Nanodevices



▸ How it works

Semiconductor single-electron pumps (courtesy PTB).

Counting flux quanta
Josephson junctions



Josephson junction:

- two superconductors coupled by a tunneling barrier
- have coupled wavefunctions

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

Under proper I_{rf} excitation amplitude of frequency f_{rf}

$$V_{\mathrm{dc}} = n\Phi_0 f_{\mathrm{rf}} = \frac{n}{K_{\mathrm{J}}} f_{\mathrm{rf}}$$

where

$\Phi_0 = h/2e = 2.067\,833\,831(13) \times 10^{-15}\,\mathrm{Wb} \quad [6.1 \times 10^{-9}]$ is the flux quantum;

$K_{\mathrm{J}} = 2e/h = 1/\Phi_0 = 483\,597.8525(30)\,\mathrm{GHz/V}$ is the Josephson constant;

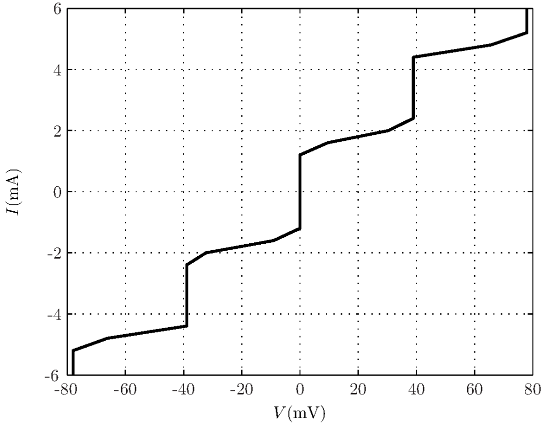
n is a small integer.

Feasible drive frequencies:

$f_{\mathrm{rf}} = 70\,\mathrm{GHz} \quad \Rightarrow \quad V_{\mathrm{dc}} = 150\,\mu\mathrm{V}$.

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

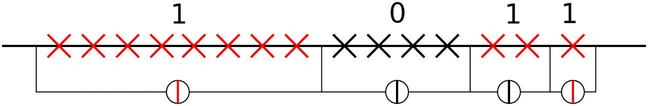


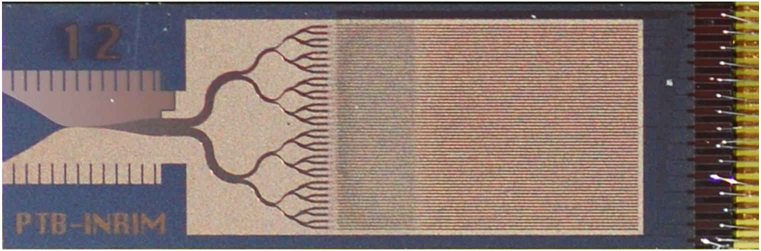
The $I - V$ characteristic of a Josephson array (256 junctions) under microwave irradiation. Steps $n = 0, \pm 1, \pm 2$ are visible. $f \approx 73\,\mathrm{GHz}$

Counting flux quanta

Josephson binary DAC

Binary-weighted Josephson DAC





Josephson junction binary array chip. 13 bit+sign DAC with 8192 superconducting-normal metal-insulator-superconductor (SNIS) junctions. The junctions are geometrically arranged over 32 parallel strips of 256 junctions each. $f = 70\,\mathrm{GHz}$. $V_{\mathrm{fullscale}} \approx \pm 1.2\,\mathrm{V}$

The quantum experiments in the framework of the present SI

Knowledge in 1989 (CODATA):

$K_J = 483\,597.9(2)\,\text{GHz/V} \qquad [4 \times 10^{-7}]$

$R_K = 25\,812.807(5)\,\Omega \qquad [2 \times 10^{-7}]$

but, *reproducibility* of Josephson and quantum Hall experiments in different experiments and different laboratories was much higher: 10^{-9}–10^{-10}

Solution: invent non-SI units! 18th CGPM resolution 6: Valid since January 1, 1990:

$K_{J\text{-}90} = 483\,597.9\,\text{GHz/V} \qquad$ [exact]

$R_{K\text{-}90} = 25\,812.807\,\Omega \qquad$ [exact]

To $K_{J\text{-}90}$ and $R_{K\text{-}90}$ the conventional units Ω_{90}, H_{90}, F_{90}, A_{90}, W_{90} are associated.
These are the electrical units in use nowadays.

The quantum experiments in the present SI

Present status of the conventional units

Becuase of improvements in the measurement of fundamental constants, today (CODATA 2014)

$K_J = 483\,597.8525(30)\,\text{GHz/V} \qquad [6.1 \times 10^{-9}]$

$R_K = 25\,812.807\,455\,5(59)\,\Omega \qquad [2.3 \times 10^{-10}]$

Therefore

$$V_{90} = 1 + 9.8(6) \times 10^{-8}\,\text{V}$$

$$\Omega_{90} = 1 - 1.764(2) \times 10^{-8}\,\Omega$$

⇒ Unacceptable deviation of the conventional units respect to the SI units

The Kibble balance

Determination of the Planck constant

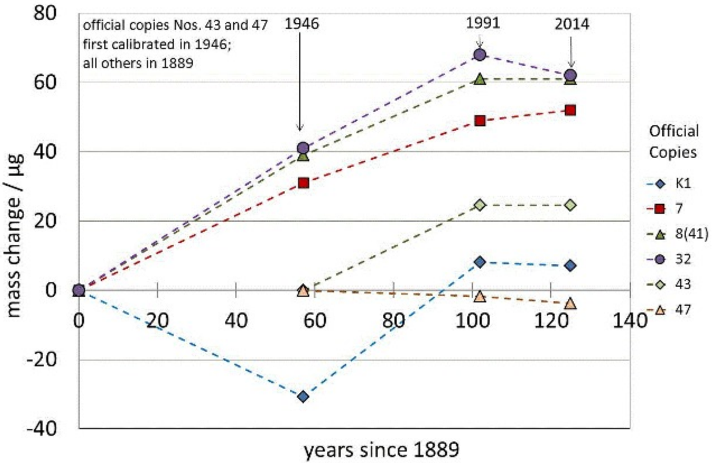
Now the derivation can be clarified

- $mgv = EI$
- $E = n\dfrac{f_\mathrm{E}}{K_\mathrm{J}}$
- $I = \dfrac{V_\mathrm{I}}{R} = \dfrac{f_\mathrm{I}}{K_\mathrm{J}}\dfrac{1}{rR_\mathrm{K}}$
- $K_\mathrm{J} = \dfrac{2e}{h}$
- $R_\mathrm{K} = \dfrac{h}{e^2}$

$$\Rightarrow mgv = hf_\mathrm{E}f_\mathrm{I}\frac{n}{r}$$

h can be measured mechanically

The problem of the kilogram



The International Prototype Kilogram compared with its *témoins*
IPK might have lost 35 µg over 130 years

The forthcoming SI

More than 10 years of discussions

INSTITUTE OF PHYSICS PUBLISHING METROLOGIA

Metrologia **42** (2005) 71–80 doi:10.1088/0026-1394/42/2/001

Redefinition of the kilogram: a decision whose time has come

Ian M Mills[1], Peter J Mohr[2], Terry J Quinn[3], Barry N Taylor[2] and Edwin R Williams[2]

The SI is the system of units in which:

- s The unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{\mathrm{Cs}}$ is 9 192 631 770 Hz;
- m the speed of light in vacuum c is 299 792 458 m/s;
- kg the Planck constant h is $6.626\,070\,15 \times 10^{-34}$ J s;
- A the elementary charge e is $1.602\,176\,634 \times 10^{-19}$ C;
- K the Boltzmann constant k is $1.380\,649 \times 10^{-23}$ J/K;
- mol the Avogadro constant N_{A} is $6.022\,140\,76 \times 10^{23}\ \mathrm{mol}^{-1}$;
- cd the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, is 683 lm/W,

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, cd, respectively, according to $\mathrm{Hz} = \mathrm{s}^{-1}$, $\mathrm{J} = \mathrm{m}^2\mathrm{kgs}^{-2}$, $\mathrm{C} = \mathrm{A\,s}$, $\mathrm{lm} = \mathrm{cd\,sr}$, $\mathrm{W} = \mathrm{m}^2\mathrm{kgs}^{-3}$.

The forthcoming SI

Redefinition of the SI base of interest for electromagnetism:

- kg the kilogram;
- A the ampere;

by fixing the values of the fundamental constants:

- *h* Planck constant;
- *e* elementary charge;

The forthcoming SI: the base unit ampere

The ampere will be redefined as:

> *The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\,176\,634 \times 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{\mathrm{Cs}}$.*

The kilogram will be redefined as:

> *The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.626\,070\,15 \times 10^{-34}$ when expressed in the unit J s, which is equal to $\mathrm{kg\,m^2\,s^{-1}}$, where the metre and the second are defined in terms of c and $\Delta\nu_{\mathrm{Cs}}$.*

The forthcoming SI: realization of the units

Consequences of the redefinition

e will be **exact**;

$\Rightarrow$ any electron-counting experiment will be a realization of the ampere;

$R_\mathrm{K} = \dfrac{h}{e^2}$ will be **exact**;

$\Rightarrow$ the quantum Hall effect will be a realization of the ohm;

$K_\mathrm{J} = \dfrac{2e}{h}$ will be **exact**;

$\Rightarrow$ the Josephson effect will be a realization of the volt;

$\Rightarrow$ Combining Josephson and quantum Hall effects with Ohm's law will be a realization of the ampere.

The forthcoming SI: electromagnetic fundamental constants

μ_0 the magnetic constant will be no more $4\pi \times 10^{-7}$ H/m: not exact and subject of measurement;

$\epsilon_0 = \frac{1}{\mu_0 c^2}$ the electric constant will be no more exact;

$\Rightarrow$ ϵ_0 and μ_0 will have the same relative uncertainty and will be totally correlated (correlation coefficient $= -1$)

$Z_0 = \mu_0 c$ the impedance of free space, and

$Y_0 = (\mu_0 c)^{-1}$ the admittance of free space will be no more exact;

h will be **exact**;

$\Rightarrow$ The Kibble balance, if traceable to K_J and R_K, will be a realization of the kilogram.

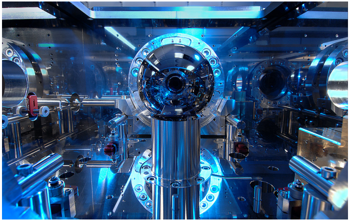
Same for the voltage and the current balances

An alternative route to realize the kilogram: silicon atom counting

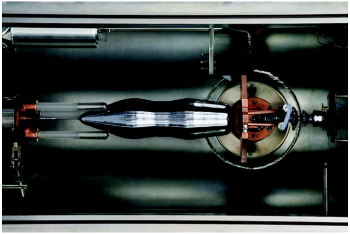
$$M_{\text{sphere}} = N \cdot m_{\text{Si}} = \frac{V_{\text{sphere}}}{v_{\text{cell}}} m_{\text{Si}}$$

Silicon atom counting

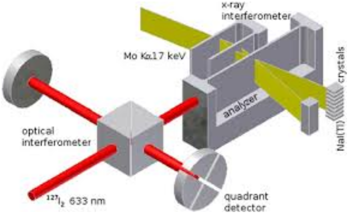
V_{sphere}: spherical interferometer



m_{Si}: single ^{28}Si crystal



v_{cell}: X-ray + optical interferometer



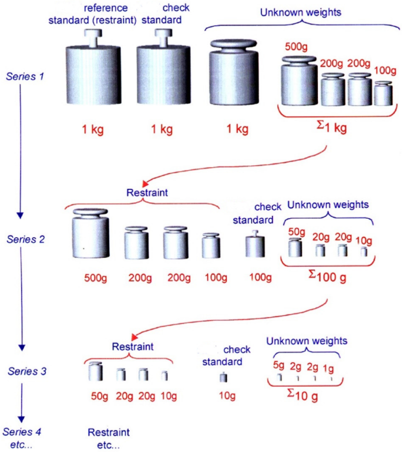
m_{Si}/h: known [10^{-9}]
from atomic experiments

$$M_{\text{sphere}} = \frac{V_{\text{sphere}}}{v_{\text{cell}}} \left(\frac{m_{\text{Si}}}{h}\right) h$$

And h is fixed in the new SI!

The forthcoming SI: benefits

Any physical experiment that satisfies the definition is a realization of the unit;



PAPER

Milligram mass metrology using an electrostatic force balance

Gordon A Shaw[1], Julian Stirling[1], John A Kramar[2], Alexander Moses[1], Patrick Abbott[1], Richard Steiner[1], Andrew Koffman[1], Jon R Pratt[1] and Zeina J Kubarych[1]

Published 28 September 2016 • © 2016 US Govt. Copyright (NIST)

Metrologia, Volume 53, Number 5

Focus on Realization, Maintenance and Dissemination of the New Kilogram

Units can be realized at any level (multiple, submultiple)

The CODATA 2017 Values of h, e, k, and N_A for the Revision of the SI*

David B. Newell[†], Peter J. Mohr[‡], Barry N. Taylor[§], and Eite Tiesinga[¶]
National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8420, USA

(Dated: July 24, 2017)

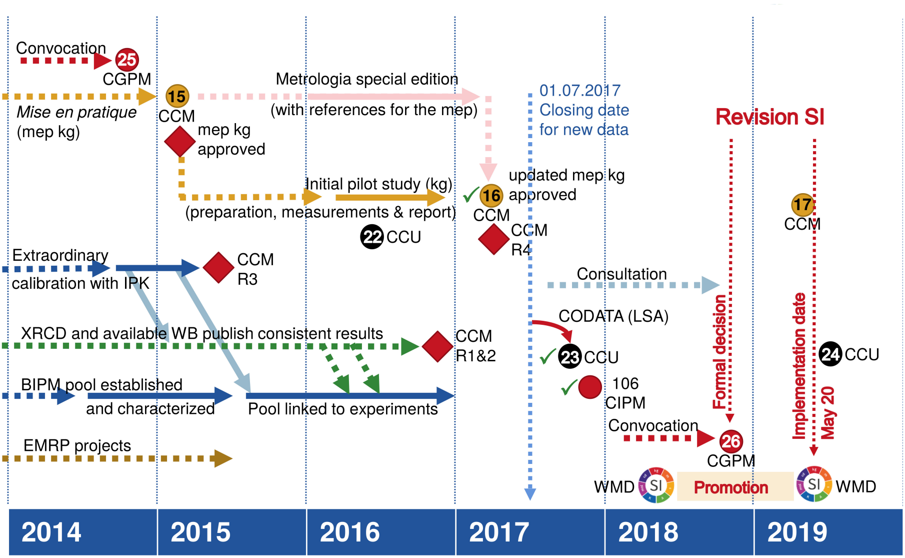
TABLE II The CODATA 2017 adjusted values of h, e, k, and N_A

Quantity	Value	Rel. stand. uncert u_r
h	$6.626\,070\,147(67) \times 10^{-34}$ J s	1.0×10^{-8}
e	$1.602\,176\,6338(81) \times 10^{-19}$ C	5.1×10^{-9}
k	$1.380\,649\,01(51) \times 10^{-23}$ J K^{-1}	3.7×10^{-7}
N_A	$6.022\,140\,761(61) \times 10^{23}$ mol^{-1}	1.0×10^{-8}

TABLE III The CODATA 2017 values of h, e, k, and N_A for the revision of the SI

Quantity	Value
h	$6.626\,070\,15 \times 10^{-34}$ J s
e	$1.602\,176\,634 \times 10^{-19}$ C
k	$1.380\,649 \times 10^{-23}$ J K^{-1}
N_A	$6.022\,140\,76 \times 10^{23}$ mol^{-1}

The roadmap towards the new SI



Formal decision: the CGPM

26th General Conference of Weights and Measures

26e CGPM
Versailles
13-16 novembre 2018

May 20, 2019

World Metrology Day

Stay prepared!

Further reading

- "Draft of the 9th SI brochure," 10 Nov 2016
- CCEM Working Group on the SI, "Mise en pratique for the ampere and other electric units in the international system of units," 2017, CCEM-17-08
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